

On Fuzzy Soft Rings

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Introduction

A soft set can be considered as an approximate description of an object. Each approximate description has two parts, namely predicate and approximate value set. The soft set is a mapping from a parameter to the crisp subset of universe.

Molodtsov [7](1999) initiated the concept of soft set theory as a new approach for modeling uncertainties. Then Maji et al. [3](2001) expanded this theory to fuzzy soft set theory. the algebraic structure of soft set theories has been studied increasingly in recent years. Aktaş and Çağman [3](2007) defined the notion of soft groups. Feng et al.[5](2008) initiated the study of soft semirings and finally, soft rings are defined by Acar et al.[1](2010). In this study, we introduce the fuzzy soft rings which is a generalization of soft rings introduced by Acar and we study some of their properties.

Fuzzy Soft Sets

Let X be an initial universe set and E be all of convenient parameter set for the universe X .

Definition

A pair (F, E) is called a soft set over X if only if F is a mapping from E into the set of all subsets of the set X , i.e., $F : E \rightarrow P(X)$, where $P(X)$ is the power set of X . [7]

In other words, the soft set is a parameterized family of subsets of the set X . Every set $F(e)$, for every $e \in E$, from this family may be considered as the set of e -elements of the soft set (F, E) , or considered as the set of e -approximate elements of the soft set. According to this manner, we can view a soft set (F, E) as consisting of collection of approximations:

$$(F, E) = \{F(e) : e \in E\}$$

Fuzzy Soft Sets

Let I be a closed unit interval, i.e., $I = [0, 1]$, and E be all of convenient parameter set for the universe X .

Definition

Let I^X denotes the set of all fuzzy sets on X and $A \subset E$.

A pair (f, A) is called a fuzzy soft set over X , where f is a mapping from A into I^X , i.e., $f : A \rightarrow I^X$. [3]

That is, for each $a \in A$, $f(a) = f_a : X \rightarrow I$, is a fuzzy set on X .

Fuzzy Soft Sets

Remark: Every fuzzy-soft-set is a soft set.

Let (F, E) be a fuzzy-soft-set, where $F : E \rightarrow I^X$ is a mapping given by $F(e) = F_e \in I^X, \forall e \in E$. Here F_e is a fuzzy set, i.e, $F_e : X \rightarrow I$.

By using the fuzzy set F_e , we can define a function $\bar{F} : A \times I \rightarrow P(X)$ such that

$$\bar{F}(e, \alpha) = (F_e)_\alpha, \quad \forall (e, \alpha) \in A \times I,$$

where $(F_e)_\alpha$ is α -level set of the fuzzy set F_e .

Hence, $(\bar{F}, E \times I)$ is a soft set. [7]

Fuzzy Soft Sets

On the other hand, if we know the soft set $(\bar{F}, E \times I)$, then by the definition of \bar{F} , we have the family of α -level sets of the fuzzy set F_{e_i} , for each $e_i \in E$. According to this manner and by using the decomposition theorem of fuzzy sets [?], we can obtain the initial fuzzy set $F_{e_i} = F(e_i)$. That is, we can reconstruct the initial function $F : E \rightarrow I^X$ and obtain the initial fuzzy soft set (F, E) . [7]

Fuzzy Soft Sets

Obviously, a classical soft set (F, E) over a universe X can be seen as a fuzzy soft set (\tilde{F}, E) according to this manner, for $e \in E$, the image of e under \tilde{F} is defined as the characteristic function of the set $F(e)$, i.e.,

$$\tilde{F}_e(a) = \chi_{F(e)}(a) = \begin{cases} 1, & \text{if } a \in F(e); \\ 0, & \text{otherwise.} \end{cases}$$

Fuzzy Soft Sets

Definition

Let (f, A) and (g, B) be fuzzy soft set over a common universe X . Then (f, A) is called a fuzzy soft subset of (g, B) and write $(f, A) \sqsubseteq (g, B)$ if

(i) $A \subset B$, and

(ii) For each $a \in A$, $f_a \leq g_a$, that is, f_a is fuzzy subset of g_a . [3]

Fuzzy Soft Sets

Definition

Let (f, A) and (g, B) be two fuzzy soft sets over a common universe X with $A \cap B \neq \emptyset$. The intersection of (f, A) and (g, B) is the fuzzy soft set (h, C) where $C = A \cap B$ and $h_c = f_c \wedge g_c, \forall c \in C$.

We write $(f, A) \sqcap (g, B) = (h, C)$. [2]

Fuzzy Soft Sets

Definition

Let (f, A) and (g, B) be two fuzzy soft sets over a common universe X . The union of (f, A) and (g, B) is the fuzzy soft set (h, C) where $C = A \cup B$ and

$$h(c) = \begin{cases} f_c, & \text{if } c \in A - B \\ g_c, & \text{if } c \in B - A, \\ f_c \vee g_c, & \text{if } c \in A \cap B \end{cases} \quad \forall c \in C.$$

We write $(f, A) \sqcup (g, B) = (h, C)$. [3]

Fuzzy Soft Sets

Definition

Let $(f_i, A_i)_{i \in J}$ be a family of fuzzy soft sets over a common universe X with $\bigcap_{i \in J} A_i \neq \emptyset$. The intersection of these fuzzy soft sets is a fuzzy soft set (h, C) where $C = \bigcap_{i \in J} A_i$ and $h(c) = \bigwedge_{i \in J} f_i(c)$, $\forall c \in C$.

We write $\bigcap_{i \in J} (f_i, A_i) = (h, C)$. [2]

Fuzzy Soft Sets

Definition

Let $(f_i, A_i)_{i \in J}$ be a family of fuzzy soft sets over a common universe X . The union of these fuzzy soft sets is a fuzzy soft set (h, C) , $C = \bigcup_{i \in J} A_i$ and for all $c \in C$,

$$h(c) = \bigvee_{i \in J(c)} f_i(c), \text{ where } J(c) = \{i \in I : c \in A_i\}. [2]$$

Fuzzy Soft Sets

Definition

If (f, A) and (g, B) are two fuzzy soft sets over a common universe X , then (f, A) **AND** (g, B) is denoted $(f, A) \tilde{\wedge} (g, B)$. $(f, A) \tilde{\wedge} (g, B)$ is defined as $(h, A \times B)$ where $h(a, b) = h_{a,b} = f_a \wedge g_b, \forall (a, b) \in A \times B$. [3]

Definition

If (f, A) and (g, B) are two fuzzy soft sets over a common universe X , then (f, A) **OR** (g, B) is denoted $(f, A) \tilde{\vee} (g, B)$. $(f, A) \tilde{\vee} (g, B)$ is defined as $(h, A \times B)$ where $h(a, b) = h_{a,b} = f_a \vee g_b, \forall (a, b) \in A \times B$. [3]

Fuzzy Soft Sets

Definition

Let (f, A) be a fuzzy soft set. The set $supp(f, A) = \{x \in A : f(x) = f_x \neq 0_x\}$ is called the support of the fuzzy soft set (f, A) .

A fuzzy soft set is said to be non-null if its support is not equal to 0_x .

Fuzzy Soft Ring

From now on, R denotes a commutative ring and all fuzzy soft sets are considered over R .

Definition

Let (F, A) be a non-null soft set over a ring R . Then (F, A) is said to be a **soft ring** over R iff $F(a)$ is a subring of R for each $a \in A$. [1]

Fuzzy Soft Ring

Definition

Let (f, A) be a non-null fuzzy soft set over a ring R . Then (f, A) is called a **fuzzy soft ring** over R iff $f(a) = f_a$ is a fuzzy subring of R for each $a \in A$, i.e.,

$$f_a(x - y) \geq f_a(x) \wedge f_a(y)$$

$$f_a(x \cdot y) \geq f_a(x) \wedge f_a(y), \forall x, y \in R.$$

That is, for each $a \in A$, f_a is a fuzzy subring in Liu's sense [2].

Example

Let N be the set of all natural numbers and define $f : N \longrightarrow I^R$ by $f(n) = f_n : R \longrightarrow I$, for each $n \in N$, where

$$f_n(x) = \begin{cases} \frac{1}{n}, & \text{if } x = k2^n, \exists k \in Z; \\ 0, & \text{otherwise.} \end{cases}, \quad \text{where } Z \text{ is the set of all integers.}$$

Then the pair (f, N) forms a fuzzy soft set over R , and the fuzzy soft set (f, N) is a fuzzy soft ring over R .

Fuzzy Soft Ring

Theorem

Let (f, A) and (g, B) be two fuzzy soft rings over R . If $(f, A) \tilde{\wedge} (g, B)$ is non-null, then it is a fuzzy soft ring over R .

Proof.

Let $(f, A) \tilde{\wedge} (g, B) = (h, AxB)$, where $h_{a,b} = f_a \wedge g_b$ for all $(a, b) \in AxB$. Since (h, AxB) is non-null $h_{a,b} = f_a \wedge g_b \neq 0_x$.

We know that $f_a, \forall a \in A$ and $g_b, \forall b \in B$ are fuzzy subrings of R and so is

$h(a, b) = h_{a,b} = f_a \wedge g_b$,

$\forall (a, b) \in AxB$, because intersection of two fuzzy subrings is also a fuzzy subring. \square

Hence $(h, AxB) = (f, A) \tilde{\wedge} (g, B)$ is fuzzy soft ring over R .

Fuzzy Soft Ring

Theorem

Let (f, A) and (g, B) be two fuzzy soft rings over R . If $(f, A) \sqcap (g, B)$ is non-null, then it is a fuzzy soft ring over R .

Fuzzy Soft Ring

Definition

Let (f, A) and (g, B) be two fuzzy soft rings over R . Then (g, B) is said to be a **fuzzy soft subring** of (f, A) if the followings are satisfied:

- (i) $B \subset A$
- (ii) g_c is a fuzzy subring of f_c , for all $c \in \text{Supp}(g, B)$.

Fuzzy Soft Ring

Theorem

Let (f, A) and (g, B) be two fuzzy soft rings over R . If $g_x \leq f_x$, for all $x \in B \subset A$, then (g, B) is a fuzzy soft subring of (f, A) .

Theorem

Let (f, A) and (g, B) be two fuzzy soft rings over R . If $(f, A) \sqcap (g, B)$ is non-null, then it is a fuzzy soft subring of (f, A) and (g, B) .

Fuzzy Soft Ring

Theorem

Let $(f_i, A_i)_{i \in J}$ be a family of fuzzy soft rings over R . Then

(i) If $\bigwedge_{i \in J} (f_i, A_i)$ is non-null, it is a fuzzy soft ring over R .

(ii) If $\bigcap_{i \in J} (f_i, A_i)$ is non-null, it is a fuzzy soft ring over R .

(iii) If $\{A_i : i \in J\}$ are pairwise disjoint, then $\sqcup_{i \in J} (f_i, A_i)$ is a fuzzy soft ring over R .

Fuzzy Soft Ideal of a Fuzzy Ring

If ℓ is an ideal of a ring R , we write $\ell \triangleleft R$.

Definition

Let (f, A) be a fuzzy soft ring over R . A non-null fuzzy soft set (γ, ℓ) over R is called *fuzzy soft ideal* of (f, A) , denoted by $(\gamma, \ell) \widetilde{\triangleleft} (f, A)$, if satisfies the following:

- (i) $\ell \subset A$
- (ii) γ_x is a fuzzy ideal of fuzzy ring $f_x, \forall x \in \text{Supp}(\gamma, \ell)$.

That is, for each $x \in \text{supp}(\gamma, \ell)$, γ_x is a fuzzy ideal in Martinez' s sense [6].

Introduction

Fuzzy Soft Sets

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Fuzzy Soft Ideal of a Fuzzy Ring

Remark: Every fuzzy soft ideal of a fuzzy soft ring (f, A) over R is a fuzzy soft subring of (f, A) , but not every fuzzy soft subring of (f, A) is a fuzzy soft ideal.

Fuzzy Soft Ideal of a Fuzzy Ring

Theorem

Let (γ_1, ℓ_1) and (γ_2, ℓ_2) be fuzzy soft ideals of a fuzzy soft ring (f, A) over R . Then $(\gamma_1, \ell_1) \sqcap (\gamma_2, \ell_2)$ is a fuzzy soft ideal of (f, A) if it is non-null.

Proof.

Let $(\gamma_1, \ell_1) \tilde{\sqsubset} (f, A)$ and $(\gamma_2, \ell_2) \tilde{\sqsubset} (f, A)$. By the definition, we write $(\gamma_1, \ell_1) \sqcap (\gamma_2, \ell_2) = (\gamma, \ell)$, where $\ell = \ell_1 \cap \ell_2$ and $\gamma_x = \gamma_1(x) \wedge \gamma_2(x)$ for all $x \in \ell$. Since $\ell_1 \subset A$ and $\ell_2 \subset A$, we have $\ell_1 \cap \ell_2 = \ell \subset A$. Suppose that (γ, ℓ) is non-null. Let $x \in \text{Supp}(\gamma, \ell)$, then $\gamma_x = \gamma_1(x) \wedge \gamma_2(x) \neq 0_x$. Since $(\gamma_1, \ell_1) \tilde{\sqsubset} (f, A)$ and $(\gamma_2, \ell_2) \tilde{\sqsubset} (f, A)$, we infer that $\gamma_1(x)$ and $\gamma_2(x)$ are both fuzzy ideals of f_x . Hence, γ_x is a fuzzy ideal of f_x for all $x \in \text{Supp}(\gamma, \ell)$. Therefore, $(\gamma_1, \ell_1) \sqcap (\gamma_2, \ell_2) = (\gamma, \ell)$ is a fuzzy soft ideal of (f, A) . □

Fuzzy Soft Ideal of a Fuzzy Ring

Theorem

Let (γ_1, ℓ_1) and (γ_2, ℓ_2) be fuzzy soft ideals of a fuzzy soft ring (f, A) over R . If ℓ_1 and ℓ_2 are disjoint, then $(\gamma_1, \ell_1) \sqcup (\gamma_2, \ell_2)$ is a fuzzy soft ideal of (f, A) .

Fuzzy Soft Ideal of a Fuzzy Ring

Theorem

$(\alpha_i, \ell_i)_{i \in J}$ be a family of fuzzy soft ideals of fuzzy soft ring (f, A) over R . Then have the followings:

- (i) $\tilde{\bigwedge}_{i \in J} (\alpha_i, \ell_i)$ is a fuzzy soft ideal of (f, A) if it non-null.
- (ii) $\prod_{i \in J} (\alpha_i, \ell_i)$ is a fuzzy soft ideal of (f, A) if it is non-null.
- (iii) If $\{\ell_i : i \in J\}$ are pairwise disjoint, then $\sqcup_{i \in J} (\alpha_i, \ell_i)$ is a fuzzy soft ideal of (f, A) if it is non-null.

Idealistic Fuzzy Soft Rings

Definition

Let (f, A) be a non-null fuzzy soft set over R . Then (f, A) is said to be an **idealistic fuzzy soft ring** over R if f_x is a fuzzy ideal of R for all $x \in \text{Supp}(f, A)$.

That is, for each $x \in \text{Supp}(f, A)$, f_x is a fuzzy ideal of R as defined by Dixit et al.

Idealistic Fuzzy Soft Rings

Theorem

Let (f, A) be a fuzzy soft set over R and $B \subset A$. If (f, A) is an idealistic fuzzy soft ring over R , then (f, B) is an idealistic fuzzy soft ring if it is non-null.

Theorem

Let (f, A) and (g, B) be two idealistic fuzzy soft rings over R . Then $(f, A) \sqcap (g, B)$ is an idealistic fuzzy soft ring over R if it is non-null.

Idealistic Fuzzy Soft Rings

Theorem

Let (f, A) and (g, B) be two idealistic fuzzy soft rings over R . If A and B are disjoint, then $(f, A) \sqcup (g, B)$ is an idealistic fuzzy soft ring over R .

Proof.

Let $(f, A) \sqcup (g, B) = (h, C)$ where $C = A \cup B$ and for all $c \in C$,

$$h(c) = \begin{cases} f_c, & \text{if } c \in A - B \\ g_c, & \text{if } c \in B - A \\ f_c \vee g_c, & \text{if } c \in A \cap B \end{cases}$$

Suppose that $A \cap B = \emptyset$. Then for all $x \in \text{Supp}(h, C)$ we have that either $x \in A - B$ or $x \in B - A$. □

Idealistic Fuzzy Soft Rings

Proof.

If $x \in A - B$, then $h_x = f_x$ is a fuzzy ideal of R since (f, A) is an idealistic fuzzy soft ring over R .

If $x \in B - A$, then $h_x = g_x$ is a fuzzy ideal of R since (g, B) is an idealistic fuzzy soft ring over R .

Thus, for all $x \in \text{Supp}(h, C)$, h_x is a fuzzy ideal of R . Consequently, $(h, C) = (f, A) \sqcup (g, B)$ is an idealistic fuzzy soft ring over R . □

If A and B are not disjoint by the related theorem, then the theorem is not true in general, because the union of two different fuzzy ideals of a fuzzy ring R may not be a fuzzy ideal of R .

Idealistic Fuzzy Soft Rings

Proof.

Thus, for all $x \in \text{Supp}(h, C)$, h_x is a fuzzy ideal of R . Consequently, $(h, C) = (f, A) \sqcup (g, B)$ is an idealistic fuzzy soft ring over R . □

If A and B are not disjoint in Theorem 5.5, then the theorem is not true in general, because the union of two different fuzzy ideals of a fuzzy ring R may not be a fuzzy ideal of R .

Idealistic Fuzzy Soft Rings

Theorem

Let (f, A) and (g, B) be two idealistic fuzzy soft rings over R . Then $(f, A) \tilde{\wedge} (g, B)$ is an idealistic fuzzy soft ring over R if it is non-null.

Theorem

Let $\varphi : R \rightarrow S$ be an epimorphism of rings. If (f, A) is an idealistic fuzzy soft ring over R , then $(\varphi(f), A)$ is an idealistic fuzzy soft ring over S .

Homomorphism of Fuzzy Soft Rings

In this section we show that the homomorphic image and pre-image of a fuzzy soft ring are also fuzzy soft ring.

Definition

Let $\varphi : X \rightarrow Y$ and $\psi : A \rightarrow B$ be two functions, where A and B are parameter sets for the crisp sets X and Y , respectively. Then the pair (φ, ψ) is called a **fuzzy soft function** from X to Y .

Homomorphism of Fuzzy Soft Rings

Definition

Let (f, A) and (g, B) be two fuzzy soft sets over X and Y , respectively and let (φ, ψ) be a fuzzy soft function from X to Y .

(1) The image of (f, A) under the fuzzy soft function (φ, ψ) , denoted by $(\varphi, \psi)(f, A)$, is the fuzzy soft set over Y defined by $(\varphi, \psi)(f, A) = (\varphi(f), \psi(A))$, where

$$\varphi(f)_k(y) = \begin{cases} \bigvee_{\varphi(x)=y} \bigvee_{\psi(a)=k} f_a(x), & \text{if } x \in \varphi^{-1}(y); \\ 0, & \text{otherwise.} \end{cases} \quad \forall k \in \psi(A), \forall y \in Y.$$

Homomorphism of Fuzzy Soft Rings

Definition

(2) The preimage of (g, B) under the fuzzy soft function (φ, ψ) , denoted by $(\varphi, \psi)^{-1}(g, B)$, is the fuzzy soft set over X defined by $(\varphi, \psi)^{-1}(g, B) = (\varphi^{-1}(g), \psi^{-1}(A))$, where $\varphi^{-1}(g)_a(x) = g_{\psi(a)}(\varphi(x))$, $\forall a \in \psi^{-1}(A), \forall x \in X$.

If φ and ψ is injective (surjective), then (φ, ψ) is said to be injective (surjective).

Homomorphism of Fuzzy Soft Rings

Definition

Let (φ, ψ) be a fuzzy soft function from X to Y . If φ is a homomorphism from X to Y then (φ, ψ) is said to be fuzzy soft homomorphism. If φ is an isomorphism from X to Y and ψ is one-to-one mapping from A onto B then (φ, ψ) is said to be fuzzy soft isomorphism.

Homomorphism of Fuzzy Soft Rings

Theorem

Let (f, A) be a fuzzy soft ring over R and (φ, ψ) be a fuzzy soft homomorphism from R to S . Then $(\varphi, \psi)(f, A)$ is a fuzzy soft ring over S .

Proof.

Let $k \in \psi(A)$ and $y_1, y_2 \in Y$. If $\varphi^{-1}(y_1) = \emptyset$ or $\varphi^{-1}(y_2) = \emptyset$ the proof is straightforward. Let assume that there exist $x_1, x_2 \in X$ such that $\varphi(x_1) = y_1, \varphi(x_2) = y_2$.

$$\begin{aligned}
 \varphi(f)_k(y_1 - y_2) &= \bigvee_{\varphi(t)=y_1-y_2} \bigvee_{\psi(a)=k} f_a(t) \\
 &\geq \bigvee_{\psi(a)=k} f_a(x_1 - x_2) \\
 &\geq \bigvee_{\psi(a)=k} (f_a(x_1) \wedge f_a(x_2)) \\
 &= \bigvee_{\psi(a)=k} f_a(x_1) \wedge \bigvee_{\psi(a)=k} f_a(x_2)
 \end{aligned}$$

□

Homomorphism of Fuzzy Soft Rings

Proof.

This inequality is satisfied for each $x_1, x_2 \in X$, which satisfy $\varphi(x_1) = y_1, \varphi(x_2) = y_2$. Then we have

$$\varphi(f)_k(y_1 - y_2) \geq \left(\bigvee_{\varphi(t_1)=y_1} \bigvee_{\psi(a)=k} f_a(t_1) \right) \wedge \left(\bigvee_{\varphi(t_2)=y_2} \bigvee_{\psi(a)=k} f_a(t_2) \right) =$$

$$\varphi(f)_k(y_1) \wedge \varphi(f)_k(y_2).$$

and similarly, we have $\varphi(f)_k(y_1 \cdot y_2) \geq \varphi(f)_k(y_1) \wedge \varphi(f)_k(y_2)$. □

Homomorphism of Fuzzy Soft Rings

Theorem

Let (g, B) be a fuzzy soft ring over S and (φ, ψ) be a fuzzy soft homomorphism from R to S . Then $(\varphi, \psi)^{-1}(g, B)$ is a fuzzy soft ring over R .

Proof.

Let $a \in \psi^{-1}(B)$ and $x_1, x_2 \in X$.

$$\begin{aligned} \varphi^{-1}(g)_a(x_1 \cdot x_2) &= g_{\psi(a)}(\varphi(x_1 \cdot x_2)) \\ &= g_{\psi(a)}(\varphi(x_1) \cdot \varphi(x_2)) \\ &\geq g_{\psi(a)}(\varphi(x_1)) \wedge g_{\psi(a)}(\varphi(x_2)) \\ &= \varphi^{-1}(g)_a(x_1) \wedge \varphi^{-1}(g)_a(x_2). \end{aligned}$$








and similarly,

we have $\varphi^{-1}(g)_a(x_1 - x_2) \geq \varphi^{-1}(g)_a(x_1) \wedge \varphi^{-1}(g)_a(x_2)$. □








References

-  U. Acar, F. Koyuncu and B. Tanay, Soft sets and soft rings, Computers and Mathematics with Applications, 59 (2010) 3458-3463.
-  B. Ahmat and A. Kharal, On fuzzy soft sets, Hindawi Publishing Corporation Advances in Fuzzy Systems (2009), Article ID 586507, 6 pages
-  H. Aktaş and N. Çağman, Soft sets and soft group, Information Science, 177 (2007) 2726 - 2735.
-  M.I. Ali, F. Feng, X. Liu and W.K.M. Shabir, On some new operations in soft set theory, Computers and Mathematics with Applications 57 (2009) 1547-1553.
-  K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 64 (2) (1986) 87-96.
-  A. Aygünoğlu and H. Aygün, Introduction to fuzzy soft groups, Computers and Mathematics with Applications 58 (2009) 1279-1286.
-  A. Aygünoğlu and H. Aygün, Soft sets and soft topological spaces, Submitted







References

-  K.V. Babitha and J.J. Sunil, Soft set relations and functions, Computers and Mathematics with Applications 60 (2010) 1840-1849.
-  N. Çağman and S. Enginoğlu, Soft matrix theory and its decision making, Computers and Mathematics with Applications 59 (2010) 3308-3314.
-  D. Chen, E.C.C. Tsang, D.S. Yeung and X. Wang, The parameterization reduction of soft set and its applications, Computers and Mathematics with Applications 49 (2005) 757-763.
-  V.N. Dixit, R. Kumar and N. Ajmal, On fuzzy rings, Fuzzy Sets and systems 49 (1992) 205-213.
-  F. Feng, Y.B. Jun and X. Zhao, Soft semirings, Computers and Mathematics with Applications 56 (2008) 2621-2628.
-  W.L. Gau and D.J. Buehrer, Vague sets, IEEE Transactions on Systems, Man and Cybernetics 23 (2) (1993) 610-614.
-  Y. B. Jun and C.H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, Information Science 178 (2008) 2466-2475.

References

-  J.L. Liu and R.X. Yan, Fuzzy soft sets and fuzzy soft groups, in: Chinese Control and Decision Conference, Guilin, China (2008)
-  W. Liu, Fuzzy invariant subgroups and fuzzy ideals, Fuzzy Sets and Systems 8 (1989) 31-41.
-  P.K. Maji, R. Biswas and A.R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9 (3) (2001) 589-602.
-  P.K. Maji, R. Biswas and A.R. Roy, Soft set theory, Computers Math. Applic. 45 (2003) 555-562.
-  P. Majumdar and S.K. Samanta, Generalised fuzzy soft sets, Computers and Mathematics with Applications 59 (2010) 1425-1432.
-  L. Martinez, Prime and primary L-fuzzy ideals of L-fuzzy rings, Fuzzy Sets and Systems 101 (1999) 489-494.
-  D. Molodsov, Soft set theory-First results, Computers Math. Applic. 37 (4/5) (1999) 19-31.

References

-  S. Nazmul and S. K. Samanta, Soft topological groups, Kochi J. Math. 5 (2010) 151-161.
-  Z. Pawlak, Rough sets, International Journal of Information and Computer Science 11 (1982) 341 - 356.
-  D. Pei and D. Miao, From soft sets to information systems, Granular Computing, 2005 IEEE International Conference on (2) (2005) 617-621.
-  A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35 (1971) 512-517
-  X. Yang, T.S. Lin, J. Yang, Y. Li and D. Yu, Combination of interval-valued fuzzy set and soft set, Computers and Mathematics with Applications 58 (2009) 521-527.
-  L.A. Zadeh, Fuzzy sets, Information and Control 8, (1965) 338 - 353.

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- Thanks**

THANK YOU !

