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ABSTRACTS

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7$^{\text{th}}$ Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

BOUNDED ORNSTEIN-UHLENBECK PROCESS FOR EXCHANGE RATE MODELLING

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In the presented work a generalization of Krugmans Target zones model [3] for the case of time-dependent solution with terminal condition has been proposed. Fixing of the terminal value of the exchange rate corresponds to joining a currency area.

In the model it is assumed that exchange rate $e(t)$ depends linearly on fundamental $f(t)$, and on expected change in the exchange rate:

$$e(\tau) = f(\tau) + \alpha \mathbf{E}_\tau[de(\tau)/d\tau], \quad \alpha > 0$$

The main assumptions of the model are that Central bank, using marginal interventions, does not allow the fundamentals to exceed the interval $[\overline{f}, \underline{f}]$, so-called "smooth pasting conditions" (see [2]): $\frac{\partial e(\underline{f})}{\partial f} = \frac{\partial e(\overline{f})}{\partial f} = 0$, and zero terminal condition: $e(T, f) = 0$.

In this work we asuume that fundamental is specified as a regulated Ornstein-Uhlenbeck process:

$$df(\tau) = -\rho(f(\tau) - \mu)d\tau + \sigma dw(\tau) - dU + dL,$$

where and ρ and σ are positive constants; μ is the long-run level of f and ρ is the speed of adjustment of the process towards μ . $dL > 0$ represents increase in money supply in case $f = \underline{f}$ and $dU > 0$ represents reductions in money supply in case $f = \overline{f}$ In this case only stationary problem has analytical solution, which is described in terms of confluent hypergeometric functions ([1]); for the non-stationary problem an analytical solution does not exist, therefore we had to use numerical methods for solving of the problem.

The proposed models can be used by monetary authorities, maintaining target zone currency policy and intending to enter a currency zone.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

ANALYSIS OF SMOOTHING PROBLEMS WITH ADDITIONAL CONDITIONS[1]

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Let X, Y be Hilbert spaces and assume that a linear operator $T: X \to Y$ and linear functionals $k_i, l_j : X \to I\!R$, $i = 1, \ldots, n$, $j = 1, \ldots, m$, are continuous. This paper deals with the following conditional minimization problems:

Problem 1 (*the smoothing problem with constraints of two-sided inequality type*)

$$\min\{\ ||Tg|| \ :\ \ |k_i g - r_i| \le \varepsilon_i,\ \ i = 1, \ldots, n,\ \ \ g \in X\ \},$$

Problem 2 (*the smoothing problem with a weight*)

$$\min\{\ ||Tg||^2 + \tfrac{1}{\omega} \sum_{i=1}^{n} (k_i g - r_i)^2 \ :\ \ \ g \in X\ \},$$

Problem 3 (*the smoothing problem with an obstacle*)

$$\min\{\ ||Tg|| \ :\ \ \sum_{i=1}^{n} (k_i g - r_i)^2 \le \delta,\ \ \ g \in X\ \}$$

for a given vector $\boldsymbol{r} = (r_1, \ldots, r_n)$ and parameters $\omega > 0$, $\delta > 0$, $\varepsilon_i > 0$, $i = 1, \ldots, n$.

We analyse these problems under the additional equality conditions

$$(1) \qquad \sum_{i=1}^{n} k_i g = \sum_{i=1}^{n} r_i \qquad \text{and/or} \qquad (2) \qquad l_j g = z_j,\ j = 1, \ldots, m,$$

which naturally appear in some practical applications ($\boldsymbol{z} = (z_1, \ldots, z_m)$ is a given vector).

Problem 1 and Problem 2 with the additional restriction (1) were investigated in [1], Problem 3 with (1) was considered in [2] and in a simplified situation in [3].

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7^{th} Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

ON SOLUTIONS OF LIENARD TYPE EQUATIONS

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We consider equation of the type

$$x'' + f(x)x'^2 + g(x) = 0, \tag{1}$$

where $f(x)$ and $g(x)$ are smooth functions. We study existence of period annuli which surround several (more than two) critical points and solutions of the Neumann boundary value problem. Our method is based on reduction to a conservative equation which is studied on a phase plane.

Let equation (1) be written as a planar system

$$x' = y, \quad y' = -f(x)y^2 - g(x). \tag{2}$$

Critical points of this system are points $(p_i, 0)$, where p_i are zeros of $g(x)$. If all zeros of $g(x)$ are simple (in the meaning that $g'(p_i) \neq 0$) then only two types of critical points are possible, namely centers and saddle points.

Recall that a critical point O of (2) is a center if it has a punctured neighborhood covered with nontrivial cycles. On the other hand, due to terminology in Sabatini [2], every connected region covered with nontrivial concentric cycles is called *a period annulus*.

A sample of results follows.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

APPLICATIONS AS THE SOURCE FOR NEW MATHEMATICAL PROBLEMS

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For the mathematicians the continual link with applications is powerful source for new statements of mathematical problems and their solutions. The concerned cooperation of specialists of particular industry and mathematicians provides successful modeling of complicated objects, phenomenon or processes. This declaration is based on several concrete mathematical examples [1; 2; 3; 4; 5].

Systems with fins and/or spines have very broad field of applications. From praxis point of view the appropriate mathematical models must be formulated as conjugated problem. In other words, the determination of temperature fields in solid system with extended surfaces can't be disconnected from the calculation of the temperature and hydrodynamic fields in the flowing around of the system media (gas or fluid). It means that the boundary conditions on the surface of the system are essentially non-homogeneous. In this talk we present approach for the determination the class of exact solutions for various systems with extended surfaces of quite complicated geometrical and thermal structure, based on the Green functions method. This approach is applicable for both steady-state and transient processes and may be enlarged for other types of mathematical models.

As the other application, the process of intensive steel quenching as the time inverse ill-posed problem for the hyperbolic heat conduction is studied. The conservative averaging method leads to the inverse well-posed problem. This problem is solved in closed form.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7[th] Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

PRICE STABILITY IN THE MARSHALL-SAMUELSON STOCHASTIC ADAPTIVE MARKET

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The paper deals with statistical model of an adaptive Samuel-Marshall type single component market. Our model supposes that a manufacturer has a monopoly there and he would like to stabilize the price of a product unit into a small neighborhood of the level $\bar{p}$. Let us remind, that in any classical single market model a price equilibrium $p(t) \equiv \bar{p}$ can be achieved by the equality of demand $D(\bar{p})$ to supply $S(\bar{p})$. To control a price at the time moment t the manufacturer can use a supplied quantity S_t, but to enter the market he needs some time $\tau(t)$, which also may be a stochastic process.Therefore manufacturer is entering the market at the moments of time t and thus he has a delayed reaction because he is guided by the price at the moment of time $t - \tau(t)$. As a result the supply S_t depends on the price $p(t - \tau(t))$. The demand D_t at the time t instantaneously affects on the price value , i.e., $D_t := D(p(t))$. As in the classical Samuelson model we will suppose equilibrium to be reached due to an adaptive price dynamical property: the price movement $(\Delta p)(t) := p(t + \triangle) - p(t)$ is proportional to difference $D_t - S_t$ multiplied by time increment $\triangle$. Practically he can operate with demand function at time moment t only as a conditional expectation of demand under condition of given price history $\mathfrak{F}^t$ up to time t analyzing statistical data $\{D_s, p(s),\, s \leq t\}$ and applying some of well derived regression procedures. Recent decades has appears many papers which intensively developed the branch of modern economics concerning the price dynamics analysis and elaboration of a rational algorithm of investor behavior, taking into account the financial market statistical uncertainty. Many researchers use Ito stochastic calculus for modeling price dynamics. As an example one can specify the well-known Black-Scholes option-pricing formula used not only by scientists in the theoretical financial economics but also by most of brokers for gambling on a stock exchange. Our paper also deals with the stochastic analysis of price dynamics, writing an adaptive Samuelson's assertion in a form of stochastic Ito equation $dp(t) = (S_t - D_t)dt + \sigma p(t) dB(t)$, where demand and supply functions were defined above, $B(t)$ is standard Brawn motion process, and parameter σ (called by *volatility*) allows to take into account value of risk connected with this model of price dynamics. The main result is formula for necessary and sufficient exponential mean square price equilibrium stability conditions involving volatility parameter, delay moment function, and elasticity demand and supply functions. Some of results have been published in [1] and [2].

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ACTA SOCIETATIS MATHEMATICAE LATVIENSIS
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

CHAOTIC MAPPINGS IN SYMBOL SPACES AND INTERVALS

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The technique of characterizing the orbit structure of a dynamical system via infinite sequences of "symbols" (in our case 0's and 1's) is known as *symbolic dynamics*.

Definition 1 ([3], [5]). The set of all infinite sequences of symbols 0 and 1 is called *the symbol space of 0 and 1* and is denoted by Σ_2, i.e.,

$$\Sigma_2 = \{s_0 s_1 s_2 ... \, | s_i = 0 \text{ or } s_i = 1, i = 0, 1, 2, ...\}.$$

The term "chaos" in reference to functions was first used in Li and Yorke's paper "Period three implies chaos" (1975). We use the following definition of R. Devaney [2]. Let (X, ρ) be metric space.

Definition 2 ([2]). The function $f : X \to X$ is *chaotic* if
a) the periodic points of f are dense in X,
b) f is topologically transitive,
c) f exhibits sensitive dependence on initial conditions.

The shift map $\sigma : \Sigma_2 \to \Sigma_2 \; \forall \; s = s_0 s_1 s_2 ... \in \Sigma_2 : \; \sigma(s) = s_1 s_2 ....$ is well known example of a chaotic map (see [3], [5] and others). But it is not an unique chaotic map in the space Σ_2. See, for example, [1]. In particular case α_m-mapping is chaotic in Σ_2.

Definition 3. The *α_m-mapping* $(m = 2, 3, ...)$ $\alpha_m : \Sigma_2 \to \Sigma_2$ is defined by

$$\alpha_m(s_0 s_1 s_2 ...) = s_1 s_2 ... s_{m-1} s_{m+1} s_{m+2} ...$$

We show that there exists semi-conjugacy between $\alpha_m : I \to I$ $(I \subset \Sigma_2)$ and corresponding class E_m of mappings in [0,1[. The topological semi-conjugacy and sensitive dependence on initial conditions guarantee that mappings E_m are chaotic.

Definition 4 ([5]). Let $f : A \to A$ and $g : B \to B$ be functions. A map $h : A \to B$ is called a *topological semi-conjugacy from f to g* provided 1) h is continuous, 2) h is onto, and 3) $h \circ f = g \circ h$. The map h is called a *topological conjugacy* if it is homeomorphism and $h \circ f = g \circ h$.

Essential result for our purpose is following:

Theorem 1 ([4]). Let A and B be subsets of the metric spaces, $f : A \to A$, $g : B \to B$, and $\tau : A \to B$ be a topological semi-conjugacy of f to g. If f is chaotic on A, then g is topologically transitive on B and has a dense set of periodic points in B. If $\tau : A \to B$ be a topological conjugacy of f and g, then f is chaotic on A if and only if g is chaotic on B.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

CRYPTOGRAPHICALLY SIGNIFICANT PROBABILYTY MEASURES

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We recall the probabilistic model of a source of plaintexts. Let A be a finite non-empty set and A^* the free monoid generated by A. The set A is also called an *alphabet*, its elements *letters* and those of A^* *finite words*. The source of plaintexts generates words $w \in A^*$.

A specific source is characterized as follows. Let $\mathcal{P} : A^* \to [0;1]$ be a map which satisfies these conditions:

(i) $\displaystyle\sum_{a\in A} \mathcal{P}(a) = 1;$

(ii) $\displaystyle\forall w \in A^* \quad \mathcal{P}(w) = \sum_{a\in A} \mathcal{P}(wa).$

Such model simulates the source of natural languages (cf. [3]). This looks like as a probability measure space. Nevertheless the question arises:

"What is the probability measure space in this case?"

We need infinite words. An (indexed) infinite word x on the alphabet A is any total map $x : \mathbb{N} \to A$. We set for any $i \geq 0$, $x_i = x(i)$ and write

$$x = (x_i) = x_0x_1 \ldots x_n \ldots$$

The set of all the infinite words over A is denoted by A^ω. The word $u \in A^*$ is called a *prefix* if there exists $y \in A^\omega$ such that $x = uy$. We denote by $\mathrm{Pref}(x)$ the set of x prefixes and for every $w \in A^*$ define the set

$$w\rangle = \{x \in A^\omega \mid w \in \mathrm{Pref}(x)\} \subseteq A^\omega\,.$$

Any set $\bigcup_{u\in\mathcal{U}} u\rangle$, where $\mathcal{U} \subseteq A^n$, is called *a cylinder of rank n*. Let $\mathcal{C}$ be the class of cylinders of all ranks. Then $\mathcal{C}$ is a set algebra and every finitely additive probability measure on the algebra $\mathcal{C}$ is in fact countable additive (cf. [1]). The extension is more or less routine in nowadays (cf. [2]). At last we identify every $u \in A^*$ with the set $u\rangle$.

I dare say this is an excellent topic for students. It demonstrates how probability, measure theory and topology works in applications. This demonstrates conections between finite and infinite mahematics too.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

MEAN SQUARE REDUCIBILITY OF LINEAR MARKOV DIFFERENCE EQUATIONS

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The behaviour of the first and second moments of the linear difference equations with Markov coefficients in $\mathbb{R}^n$:

$$x_t = A(\xi_t)x_{t-1} \tag{1}$$

solution is analyzed. Where $\{\xi_1, t \in \mathbb{Z}\}$ is a homogeneous Markov Feller chain with phase metric compact space $\mathbf{Y}$ and transition probabilities $P(y, dz)$. For the analysis of the first moments a linear continuous operator in a space of continuous n-dimensional reproductions is defined

$$y \in \mathbf{Y}, u \in \mathbb{C}_n(\mathbf{Y}) : (\mathbf{A}u)(y) := \int\limits_{\mathbf{Y}} A^T(z)u(z)P(y, dz) \tag{2}$$

The possibility of (1) mean reducibility is considered in the case when matrix function $\{A(y), y \in \mathbf{Y}\}$ is near to constant and can be given in a form $A(y) = A_0 + \varepsilon \sum\limits_{k=0}^{\infty} \varepsilon^k A_{k+1}(y)$, where $\varepsilon \in (0, 1)$. Therefore the operator (2) also can be expressed [1],[2] as $\mathbf{A}(\varepsilon) = \mathbf{A}_0 + \varepsilon \sum\limits_{k=0}^{\infty} \varepsilon^k \mathbf{A}_{k+1}(\varepsilon)$, hereby the operator $\mathbf{A}_0$ can be represented as an operator tensor product $\mathbf{A}_0 = \mathcal{P} \otimes A_0$, where $\mathcal{P}$ is a Markov operator. As main assumption for mean reducibility of the equation (1) is $\sigma(A_0) \cap \sigma_\rho = \emptyset$, where $\sigma_\rho = \{\lambda_1\lambda_2 : \lambda_1 \in \sigma(\mathcal{P}) \setminus \{1\}, \lambda_2 \in \sigma(A_0)\}$.

Lemma 1. *If all above mentioned assumptions are into force, then for sufficiently small* $\bar{\varepsilon} > 0$ *and all* $|\varepsilon| < \bar{\varepsilon}$ *a difference equation is mean reducible, hereby the matrix function* $\{B(y, \varepsilon), y \in \mathbf{Y}\}$ *is a basis in operator* $\mathbf{A}(\varepsilon)$ *root subspace which corresponds to the spectrum* $\sigma_0(\varepsilon)$ *part which is defined by equality* $\lim\limits_{\varepsilon \to 0} \sigma_0(\varepsilon) = \sigma(A_0)$. *For each* $|\varepsilon| < \bar{\varepsilon}$ *matrices* $\{B(y, \varepsilon), y \in \mathbf{Y}\}$ *and* $\Lambda(\varepsilon)$ *unambiguously are defined by equality*

$$y \in \mathbf{Y}, |\varepsilon| < \bar{\varepsilon} : (\mathbf{A}(\varepsilon)B)(y, \varepsilon) = B(y, \varepsilon)\Lambda^T(\varepsilon) \tag{3}$$

where matrix $\Lambda(\varepsilon)$ *is operator* $\mathbf{A}(\varepsilon)$ *restriction matrix to corresponding root subspace.*

Solving the equation (3) the decompositions of matrices $\{B(y, \varepsilon), y \in \mathbf{Y}\}$ and $\Lambda(\varepsilon)$ can be found. For dynamics of the second moment matrix of difference equation (1) solution, behavior of matrix $Q_t := \mathbb{E}\{x_t x_t^T\}$ should be investigated. For matrices sequence $(xx)_t := x_t x_t^T$ a linear difference equation in space of symmetric matrix-functions $\mathbb{M}_n$ can be written as $(xx)_t = A(\xi_t)(xx)_{t-1}A^T(\xi_t)$. The obtained results from the first moment analysis can be adapted to the analysis of a linear continuous operator $(\mathbf{A}q)(y) := \int\limits_{\mathbf{Y}} A^T(z)u(z)A(z)P(y, dz)$.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

INTEGRAL CONTINUITY AND THE LAW OF LARGE NUMBERS FOR FAST OSCILLATED RANDOM EVOLUTION

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The paper deals with two-parametric families $T(t,s), t \geq s \geq 0$ of linear continuous operators acting in Banah space $\mathbf{V}$ and satisfying an equalities $T(t,u)T(u,s) = T(t,s)$ for all $t \geq u \geq s \geq 0$, and $T(t,t) = I$ for all $t \geq 0$. Under assumption that above operators may be presented as a shift operator family defined by linear differential equation $dv(t) = \mathcal{A}(y(t/\varepsilon))v(t)dt$ in $\mathbf{V}$ with right part dependent on ergodic Markov process $y(t)$ and small positive parameter ε we will discuss a possibility for problem simplification of the above dynamical system applying the low of large numbers in an integral form to operator family $\mathcal{A}(y(t))$. If the above Markov process is ergodic with unique invariant measure $\mu(dy)$ we will approximate the above two-parametric semigroup with continuous operator semigroup $\bar{T}(t)$ on $\mathbf{V}$ generated by averaged operator $\bar{\mathcal{A}} := \int \mathcal{A}(y)\mu(dy)$ and to prove that under assumption of uniform convergence of averaging procedure one can apply an *integral continuity property* in a following form

$$\forall\delta > 0, \exists\varepsilon_0 > 0, \forall\epsilon \in (0,\varepsilon_0): \quad \mathbb{P}\left\{\max_{0\leq t\leq T}\left|\left|\int_{t_0}^{t_0+t}[\mathcal{A}\left(\frac{s}{\varepsilon}\right) - \bar{\mathcal{A}}]\varphi ds\right|\right| \leq \delta\right\} = 1$$

uniformly on $T > 0, s \geq 0$ and $\varphi \in \{\mathbf{V} : ||\varphi|| \leq 1\}$. The above inequality permits to establish a closeness of operator families $T(t,s)$ and $\bar{T}(t)$ on time-interval $0 \leq t \leq \frac{T}{\varepsilon}$ for any $T > 0$ and sufficiently small $\varepsilon > 0$. Furthermore while $\varepsilon > 0$ is sufficiently small the above operator families have the same exponential asymptotic and close indices when $t \to \infty$. The same assertions may be formulated also for periodic or semi-periodic nonrandom $y(t)$. These results have been applied to asymptotical analysis of stochastic differential equations with coefficients dependent on Markov process and to impulse differential equations with Markov switchings. For that we have used an averaging procedure to infinitesimal operators of enlarged Markov processes defined by above random dynamical systems. Our proposal method may be also applied for qualitative analysis of linear stochastic functional differential equations with small or rapidly oscillating perturbations dependent on phase coordinates and an ergodic Markov process. We have constructed an algorithm for dynamical analysis of the initial random equation with delay that permits approximate its solutions by corresponding solutions of an averaged deterministic ordinary functional differential equation. It has been proved that problem of stability analysis of linear retarding dynamical systems with rapidly oscillating functionals one may reduce to stability analysis of deterministic linear functional differential equation applying an averaging procedure to above mentioned functionals. Some of reported results have been published in [1].

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

REŽĢU KRIPTOGRĀFIJA

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Par režģi m-dimensionālā Eiklīda telpā $\mathbb{R}^m$ sauksim n lineāri neatkarīgu vektoru $b_1, \ldots, b_n$, $b_i \in \mathbb{R}^m$ lineāro kombināciju pa veseliem skaitļiem kopu

$$\mathcal{L}(b_1, \ldots, b_n) = \left\{ \sum_{i=1}^{n} x_i b_i : x_i \in \mathbb{Z} \right\},$$

kur $m \geq n$. Vektoru virkni $\{b_1, \ldots, b_n\}$ sauc par režğa bāzi.

Aplūkosim režğu kriptogrāfisko lietojumu duālo dabu - ar tiem saistītas NP-pilnas problēmas un uz tām balstītus šifrēšanas algoritmus, kā arī tuvinātas šo problēmu risināšanas metodes un to lietojumus kriptoanalīzē.

Acta Societatis Mathematicae Latviensis
Abstracts of the 7^{th} Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

SOME SOLVED AND UNSOLVED PROBLEMS OF COMBINATORIAL GEOMETRY

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A few unsolved problems of combinatorial geometry are discussed. Attention has been focused also on pupils and students findings.

Lattice convex polygons with minimum area [1; 2; 3].

Let $a(n)$ be a minimum area of a convex polygon whose all n vertices are points on the integer lattice. The function $a(n)$ has been studied by many authors. For general n, only lower and upper bounds have been obtained, but for "small" n the values of $a(n)$ have been determined for $n = 2k, k \leq 11$, and for all $n \leq 10$ [1; 2]. Several new improvements of upper bounds of $a(n)$ (may be sharp) are obtained.

"**Eternity II** is the puzzle with the $2 million prize up for grabs which launched world-wide on 28th July 2007. The puzzle consists of 256 square pieces that are bordered by coloured patterns which must be aligned across the whole puzzle. Unlike most puzzles, which only have one correct way of completing the final solution, there are thousands of ways that Eternity II can be solved to win the $2 million prize." [4]

Pentomino twins problem: *find all pentomino twins fitted on a chess board* 8×8. This attractive and previously unsolved problem has been studied by I. Saknīte, and M. Virza (Valmiera State Gymnasium, Form 11, 2006) in their contest work. They were nominated to participate in the 18th European Union Contest for Young Scientists, Stockholm. M. Virza was able to create computer programmes by means of which one can find all pentomino twins.

Compatibility of polyforms. Some new striking results [5; 6; 7].

How to find the (smallest) region that can be tiled independently by two different polyforms A and B? Polyforms A and B are said to be compatible if such region exists.

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ACTA SOCIETATIS MATHEMATICAE LATVIENSIS
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

SIMULTANEOUS CONFIDENCE BANDS FOR QUANTILE-QUANTILE AND PROBABILITY-PROBABILITY PLOTS

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Suppose we have independent and identically distributed random variables samples $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ with unknown functions F_1 and F_2 respectively. Our goal is to check the following hypothesis

$$H_0 : F_1 = F_2 \quad \text{against} \quad H_1 : F_1 \neq F_2.$$

There exists a lot of test procedures in statistical literature dealing with this problem. Most famous are: Kolmogorov-Smirnov, Cramér-von Mises and Anderson-Darling tests. An alternative method is to construct simultaneous confidence bands for quantile-quantile (Q-Q) or probability-probability (P-P) plots which are defined as

$$F_1 F_2^{-1}(t), t \in (0, 1)$$

and

$$F_1^{-1} F_2(x), x \in \mathbb{R}.$$

We analyze the asymptotic behavior of the empirical P-P and Q-Q plot processes defined respectively as

$$\Delta_{n,m}(t) := \sqrt{n}(F_{1n} F_{2m}^{-1}(t) - F_1 F_2^{-1}(t)), t \in (0, 1),$$

$$\delta_{n,m}(x) := \sqrt{n} f_1(F_1^{-1} F_2(x))(F_{1n}^{-1} F_{2m}(x) - F_1^{-1} F_2(x)), x \in \mathbb{R},$$

where F_{1n} and F_{2m} are empirical distribution functions and f_1 is the density function of F_1 (see, for example, [2], [3]). To construct simultaneous bands we need to know the asymptotic limiting distribution of supremum of $\Delta_{n,m}$ and $\delta_{n,m}$. Beirlant and Deheuvels [2] analyze these processes under the null hypothesis $F_1 = F_2$. In general the limiting distributions of $\Delta_{n,m}$ and $\delta_{n,m}$ are functions of Brownian Bridges and unknown distribution functions F_1 and F_2 which have to be estimated by some non-parametric method such as the kernel smoothing method (see [1]).

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

ON VENN DIAGRAMS

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This is a short survey of some facts, related to logic, geometry and combinatorics, about Euler and Venn diagrams.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

ALGEBRA AUTOMORPHISM ACTION IN THE TAME CASE

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The classic module theory of associative algebras studies the module categories over algebras with fixed algebra structures. Modules are usually studied up conjugation action of the general linear group of the underlying vector space of the module. An important problem in module theory is the classification of indecomposable modules over an algebra up to isomorphism. It is known that algebras can be divided into three classes depending on the complexity of their module categories - finite, tame and wild representation type. The module classification problem have been solved in many cases for the finite and tame representation type.

In [2] we introduced an additional group action on the category of modules - the action of the automorphism group of the algebra. The orbits of this action define an equivalence relation which is coarser than the standard module isomorphism. This equivalence relation allows us to study representations of the algebra up to algebra automorphisms. The defined equivalence relation can contribute to the understanding of classic module categories in the tame and wild cases, in particular, some parameters of multiparameter families of indecomposable modules can be attributed to this action.

Another research direction could be related to a characterization of tame algebras using algebra automorphism action.

In [2] we showed that the one-parameter families of finitely generated indecomposable modules over $k[X]$ and $k[X]$, $k[X,Y]/(X,Y)^2$ are orbits under the algebra automorphism action.

Here we present a similar results concerning two families of local finitely generated algebras of tame representation type - dihedral algebras $k[X,Y]/(X^2,Y^2,(XY)^kX^{\epsilon_1},(YX)^kY^{\epsilon_2})$, $\epsilon_i \in \{0,1\}$ and semidihedral algebras $k[X,Y]/(X^3,Y^2,X^2-(YX)^kY)$. These results are based on the known clasifications of indecomposable modules for these algebras given in [1] and [3].

Proposition 1. *The one-parameter families of indecomposable finitely generated modules over dihedral and semidihedral algebras belong to a single orbit under the algebra automorphism action.*

Proof. For each one-parameter family of modules we exhibit a family of algebra automorphisms which transforms modules into a canonical representative of the module family. □

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

A EU FINANCED PROJECT FOR MATHEMATICS TEACHING IN REZEKNES AUGSTSKOLA

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We report on a EU co-financed ESF project "Modernization of methodical and technical support for mathematics studies in engineering study programs in Rezeknes Augstskola", Nr. 2006/0256-/VPD1/ESF/PIAA/06/APK/3.2.3.2/0100/0160.

In order to stimulate development of Latvian economy it is important to improve quality of engineering study programs in Latvian universities since high quality engineering education enhances development of producing branches of the economy, increases employment and improves competetiveness. Mathematics plays an important role in all engineering disciplines as a unifying language and a universal body of knowledge.

The overall goal of the project is to improve the quality of mathematics teaching in engineering study programs in Rezeknes Augstskola by modernizing the computing and presentation equipment and developing new teaching resources.

We describe the main goals, activities and the results of the project.

Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

BLACK-SHOLES FORMULA FOR BS-MARKET WITH FAST OSCILLATING VOLATILITY

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In Black-Sholes continuous-time European style option on a stock valuation framework, the stock price S is assumed to obey geometrical Brownian motion

$$dS(t) = \alpha(t)S(t)dt + \sigma(t)S(t)d\omega(t). \tag{1}$$

If $\alpha(t)$ and $\sigma(t)$ are constants closed-form valuation formulas for European style options can be derived. In our work we assume that volatility $\sigma = \sigma(\xi(y))$ follows some Markov process $\xi(y)$ so that it has jumps at random time moments τ_j but it is constant between jumps. $\{\tau_j\}$ are switching moments of Poisson process. Under these assumptions we define the following impulse system

$$\frac{d\sigma(t)}{dt} = 0 \tag{2}$$

$$\sigma(\tau_j) = \sigma(\tau_j-) + \varepsilon b(\sigma(\tau_j-), \xi(\tau_j-)) \tag{3}$$

Further we make assumptions that volatility σ is a fast oscillating process with small jumps (it allows ε in equation (3) to be a small positive number). Assuming that changes of the volatility on average are close to zero and a few additional conditions are met we apply diffusion approximation methodology developed in [1] and [2]. We are able to show that our model converges weakly to the solution of continuous time stochastic diffusion equation of the following type

$$d\sigma(t) = k\sigma(t)dt + \theta\sigma(t)d\omega_1(t). \tag{4}$$

This result lets us to use closed-form option valuation formulas derived in [3] to value European style currency options if the volatility of the exchange rate follows (4).

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7$^{\text{th}}$ Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

ON A CLASS OF SIXTH ORDER ORDINARY DIFFERENTIAL EQUATIONS AND RELATED PROBLEMS

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We consider linear equations of the form

$$x^{(6)} = p(t)x, \tag{1}$$

where $p(t)$ is continuous positive valued function. A set of results concerning oscillatory behavior of solutions of equation (1) is provided. Special attention is paid to solutions of equation (1) with a quadruple zero at $t = a$.

A theorem on multiplicity of solutions of a nonlinear boundary value problem

$$x^{(6)}(t) = f(t, x(t)), \tag{2}$$

$$x(a) = x'(a) = x''(a) = x'''(a) = 0, \tag{3}$$

$$x(b) = x'(b) = 0, \tag{4}$$

is stated.

Our principal result consists of estimation from below of the number of solutions to the boundary value problem (2), (3), (4) provided that it has at least one solution $\xi(t)$. This estimate depends on the oscillatory behavior of the equation (1).

Our method of proof is similar to that used by F. Sadyrbaev [4] for the case of 4th order nonlinear equation and is based on representation of the nonlinear equation (1) as a family of linear equations, the coefficients of which depend on solutions of (1) satisfying the initial value conditions (3).

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

IMPACT OF LEARNING MATHEMATICS ON THE PERSONALITY DEVELOPMENT

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Impact of learning Mathematics on the personality development is generally recognised; however it is difficult to describe it, because neither psychology nor pedagogy presents a unified theory about the personality structure. Most of the researchers admit that personality development is determined by three factors and their interaction. These factors are natural aptitude or heredity, impact of the environment (mainly - social environment) and the activity of the personality himself/herself.

Acquisition of Mathematics is a constituent part of education, but education is one of the manifestations of the social environment. Impact of the natural aptitude and the person's own activity in the educational field are characterised by the emotional, physical, volitional and intellectual peculiarities of the personality. The process of learning Mathematics is accompanied by such emotions as the feeling of the beauty, harmony and the innovative, wonder and curiosity, zeal and confidence in the correctness of the achieved results, feeling of doubt and satisfaction when learning mathematics. Mathematicians usually stress the positive emotions, which arise from doing mathematics. The American mathematician founder of cybernetics Norbert Wiener characterised his feelings in a very interesting way (1894 - 1964): "Almost each of my emotional experiences always, to some or another degree, reflects one or another mathematical situation, which is not clear to me as yet or has not managed to express itself through certain formulas". "I do not know, what I may appear to the world, but to myself I to seem to have been a small boy, playing on the sea-shore, and diverting myself in now and then finding a colourful pebble or a rosy shell than ordinary, whilst the great ocean of truth lay all undiscovered before me" that is what Isaac Newton (1643 -1727), a mathematician and physicist, has acknowledged. The feeling of beauty helps mathematicians-scientists in their work of. The American and German mathematician Hermannn Weyl (1885 - 1955) wrote: "In my work I have always been trying to combine the truth with the beautiful and when I had to make my choice among two things, I always chose the beautiful". Manifestations of one's will is the ability and readiness to always overcome difficulties, purposefulness, perseverance, love of work and initiative, they all are closely linked with the structure of the personality needs and the field of motivation. Physical manifestations needed for learning mathematics are the ability to work and indefatigability. Intellectual manifestations are the ability to think in an algorithmic way and logically, as well as spatial imagination.

At present an urgent issue in mathematical studies is the insufficient number of secondary school leavers willing to study and being able to successfully complete study programmes in engineering sciences and exact sciences.

Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

NONLINEAR SPECTRA FOR FUČIK TYPE PROBLEMS WITH THE NEUMANN BOUNDARY CONDITIONS

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Eigenvalue problems of the form $x'' = -\lambda f(x^+) + \mu g(x^-)$, (i), $x'(a) = 0,\ x'(b) = 0$, (ii) are considered. We are looking for (λ, μ) such that the problem $(i), (ii)$ has a nontrivial solution. This problem generalizes the famous Fuchik problem for piece-wise linear equations. In our considerations functions f and g may be nonlinear. Consequently spectra may differ essentially from those for the Fučik equation.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

A PRIORI ESTIMATES OF THE BOUNDARY VALUE PROBLEM FOR THE SECOND ORDER DIFFERENCE EQUATIONS

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It is often in literature used a priori estimates in terms of the lower and upper functions for the boundary value problems of the second order difference equation

$$x(n+2) - 2x(n+1) + x(n) = f(n, x(n))$$

as well as corresponding equations on time scales (for example [1]). The aim of the present report is to attract the attention in order to obtain a priori estimates for the difference $x(n+1) - x(n)$ of the solution $x(n)$ having a priori estimates in the case of the more general difference equation with continuous nonlinearity

$$x(n+2) = g(n, x(n), x(n+1)).$$

Particularly, the existence theorem in the case of Dirichlet type boundary conditions is proposed. The possible extensions of the formulated results will be discussed.

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ACTA SOCIETATIS MATHEMATICAE LATVIENSIS
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

ON 4th MOMENT STATIONARY GARCH(p, q) PROCESS

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Unlike other sub branches of modelling, mathematic modelling enables the researching only of those object-original parameters, which can be described numerically and included in equations and systems of equations, i.e., formalized. To a certain extent, that diminishes practical significance of mathematical modelling. However, alongside with purely practical advantages (less consumption of materials, remarkable time economy, number and quality of analytical calculations, etc.) there are many other advantages, which move it forward as an indispensable scientific research method.

In the process of mathematical modelling, the researched object can be described determinately and stochastically. There are different statistical characters (the conditions of the object functioning probabilities), applied for the description of the object with stochastical (probability) models. The statistical characters are obtained with processing of data accumulated in the process of object provision or data of the experimental research without detailed research of the inner mechanism, which could explain the reasons for the detected processes. The mathematical models are created and applied to develop a forecast for the future cognition of the object. However, admittedly the credibility of the forecast is not always being researched in sufficiently detailed way. There are cases, when conventional forecast methods give faulty forecasts.

Homoscedastity is one of the unmanageable preconditions of the regression analysis. This means that with the change of factorial feature values the outcome feature dispersion must not systematically change. If this requirement has not been satisfied, the division of statistical set is heteroscedastic. The target of this paper is to evaluate the conditions at which it is possible to apply heteroscetastic regression models in management forecasting and decision taking processes.

The practical application of the results connected with the development of GARCH model and analysis of any time series and use of the obtained results for the assessment of its forecasting possibilities. The main result is GARCH(p,q) type criterion of autoregression models with heteroscedastic residue and algorithm for stability and stationary analysis. The obtained results are based on Markov's method of linear dynamic systems covariation analysis.

Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

SINGULAR SPECTRAL ANALYSIS USE FOR FILTERING TASKS SOLVING

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There are many practical problems related to signal extraction from the mixture "signal plus noise" [1]. From the practical point of view the main task of filtering theory is to provide data processing methods for signal identification under different assumption about noise statistical properties. At present time the many such methods are worked out, starting from the simple Moving Averages and optimal Kalman filters [2] up to very complicated long-memory adaptive spectral and neural networks filters. The common feature of all such methods is the following - for efficient filtering it is necessary to know the statistical properties of noise.

Singular spectral analysis (SSA) is one of the best methods, which allows to separate signal form noise [3] under very poor knowledge about noise statistical properties [4]. The base scheme of SSA consists of four steps.

The first step is called the embedding step and converts one-dimensional time series into L-dimensional vectors $X_i = (f_{i-1}, \ldots, f_{i+L-2})^T, 1 \leq i \leq K$.

The second step is called the singular value decomposition (SVD) step and represents trajectory matrix as the sum or resultant matrices $\mathbf{X} = \mathbf{X}_1 + \ldots + \mathbf{X}_d$, where $\mathbf{X}_i = \sqrt{\lambda_i} U_i V_i^T, U_1, \ldots, U_L$ are eigenvectors corresponding to eigenvalues $\lambda_1, \ldots, \lambda_L, V_i = \frac{1}{\sqrt{\lambda_i}} \mathbf{X}^T U_i$.

The third step is called the grouping and distributes the eigenvalues $\lambda_1, \ldots, \lambda_L$ into m subsets $I_i, \ldots, I_m$.

The fourth step is called as diagonal averaging and calculates new averaged values of resultant matrices for each $I_i, \ldots, I_m$, representing initial time series as sum of m series.

The basic capabilities of SSA include possibility to extract trends of different resolutions, perform smoothing, extract cyclic components with small and large periods, and extract periodicities with varying amplitudes. Those capabilities were used for several practical filtering tasks solving, such as leaks identification in pipelines, when it is necessary to filter small pressure variations under intensive technological noise, and financial time series forecasting, when it is necessary to filter regular events under intensive market noise.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7$^{\text{th}}$ Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

ANALYTIC-NUMERICAL SOLUTIONS OF NEWTON PROBLEM FOR 3D PARABOLIC DIFFERENTIAL EQUATION

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Newton problem for the parabolic differential equation is formulated as follows

$$\frac{\partial}{\partial t}c\left(x,t\right)=D\sum_{n=1}^{3}\frac{\partial^2}{\partial x_n^2}c\left(x,t\right)+\sum_{n=1}^{3}v_n\frac{\partial}{\partial x_n}c\left(x,t\right)+f\left(c,t,x\right), \tag{1}$$

$$x=(x_1,x_2,x_3):0<x_n<l_n, n=\overline{1,3}, t>0;$$

$$\gamma_{n1}\left(c\left(x,t\right)\right)_{|x_n=0}+\gamma_{n2}\left(\frac{\partial}{\partial x_n}c\left(x,t\right)\right)_{|x_n=0}=c_n\left(x\backslash\{x_n\},t\right), n=\overline{1,3}, \tag{2}$$

$$0\le x_m\le l_m, m=\overline{1,3}, m\neq n, t\ge 0;$$

$$\gamma_{n3}\left(c\left(x,t\right)\right)_{|x_n=l_n}+\gamma_{n4}\left(\frac{\partial}{\partial x_n}c\left(x,t\right)\right)_{|x_n=l_n}=c_{3+n}\left(x\backslash\{x_n\},t\right), n=\overline{1,3}, \tag{3}$$

$$0\le x_m\le l_m, m=\overline{1,3}, m\neq n, t\ge 0;$$

$$c(x,0)=c_0(x), 0\le x_n\le l_n, n=\overline{1,3}. \tag{4}$$

General solution of the problem (1)-(4) is known, but in this work, in order to fulfil solution existence conditions, for this Newton problem we show the following new limitations, expressed as bilateral inequalities [1] joining the velocities v_n $(n=\overline{1,3})$ to the γ_{nm} $(n=\overline{1,3}; m=\overline{1,4})$:

$$-2D\frac{|\gamma_{n1}|}{\gamma_{n2}}\le v_n\le 2D\frac{\gamma_{n3}}{\gamma_{n4}}, \gamma_{n2}\neq 0,\neq\ \gamma_{n4}\neq 0, n=\overline{1,3}. \tag{5}$$

Programme code algorithm is developed for finding eigenvalues from transcendental equation by using Newton interval method. Analytical solution of the Newton problem based on Green function approach is proposed. Numerical implementation of solution is realized. On the base of test examples the numerical solution accuracy is estimated, including the cases, when limitations (5) are violated. The crucial role of limitations (5) was shown, avoiding which the solution becomes divergent.

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ACTA SOCIETATIS MATHEMATICAE LATVIENSIS
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

DATORPROGRAMMU LIETOŠANA PARCIĀLO DIFERENCIĀLVIENĀDOJUMU RISINĀŠANĀ

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Sakarā ar ESF projektu "Datoru matemātisko sistēmu ieviešana mācību procesā augstskolā", Nr. VPD1/ESF/PIAA/06/APK/3.2.3.2/0093/0063 (1.11. 2006.– 20.08. 2008.) ir iegādātas un tiek ieviestas matemātikas mācību procesā 3 Latvijas lielākajās universitātēs — LU, RTU, LLU, Ventspils un Rēzeknes Augstskolās mūsdienu universālās datoru matemātiskās sistēmas (DMS), t.i. datorprogrammu (DP) MATLAB, MAPLE un MATHEMATICA licenses.

Matemātiskajā modelēšanā DMS līderis pasaulē ir DP-a MATLAB (versiju 7.2, ko izstrādājusi MathWorks firma ASV). Simboliskos un analītiskos pārveidojumos DMS līderi ir MAPLE un MATHEMATICA (versija 10 "Maplesoft firma Kanādā", un 5.2 " Wolfram Research firma ASV").

Jaunākajās DP versijās ir iespējams risināt parasto diferenciālvienādojumu (DV) robežproblēmas, kā arī jaukta veida problēmas parciālajiem diferenciālvienādojumiem (PDV).

Apskatīsim nelineāru jaukta veida problēmu paraboliska tipa PDV (MATLAB pieraksts):

PDV: $\mathbf{c(x,t,u_x)\,u_t = x^{-m}(x^m f(x,t,u_x))_x + s(x,t,u_x), u = u(x,t), xk < x < xl, t > t0;}$

Robežas nos.: $\mathbf{pk(xk,t,uk) + qk(xk,t)f(xk,t,uk_x) = 0,\ pl(xl,t,ul) + ql(xl,t)f(xl,t,ul_x) = 0;}$

Sākuma nos.: $\mathbf{u(x,t0) = u0(x).}$

Piemērā: $\mathbf{xk = 0, xl = 1, qk = ql = 0, pk = uk, pl = ul(u(0,t) = 0, u(1,t) = 0), u0(x) = x(1-x),}$ $\mathbf{m = 0, c = 1, f = (4u^3)u_x, s = 5u^3.}$

1) MATHEMATICA - 5.2 komandas (atverot iekavas):

$\mathbf{atr = NDSolve\,[\{Derivative[0,1][u][x,t] = 12 * u[x,t]^2 * (Derivative[1,0][u][x,t])^2 +}$
$\mathbf{4 * u[x,t]^3 * Derivative[2,0][u][x,t] + 5 * u[x,t]^3\},}$
$\mathbf{\{u[0,t] = 0, u[1,t] = 0, u[x,0] = x * (1-x)\}, u, \{x,0,1\}, \{t,0,10\}]}$
$\Rightarrow \{\{u-> InterpolatingFunction[\{\{0,1\},\{0,10\}\},?]\}\}$
$\mathbf{Plot3D\,[Evaluate[u[x,t]/.atr[[1]], \{t,0,10\}, \{x,0,1\}, PlotRange \to All]}$
$\Rightarrow$ zīmē krāsainu virsmu.

2) MAPLE - 10 komandas (iekavas atver automātiski):

$\mathbf{PDV := diff\,(u(x,t),t) = diff\,(u(x,t)^4, x, x) + 5 * u(x,t)^3 :}$
$\mathbf{Nos := \{u(x,0) = x * (1-x), u(0,t) = 0, u(1,t) = 0\} :}$
$\mathbf{atr := dsolve\,(PDV, Nos, numeric);}$
$\Rightarrow$ atr:= module() export, plot, plot3D, animate, value, setting: end module
$\mathbf{atr : -plot3d\,(t = 0..10, x = 0..1, axes = boxed);}$ $\Rightarrow$ zīmē krāsainu virsmu.

3) MATLAB - 7.2 komandas, kuras ieiet M- failā **konf_LMB:**

$\mathbf{m = 0; x = linspace\,(0,1,100); t = linspace\,(0,10,20);}$
$\mathbf{atr = pdepe\,(m, @PDV, @SNos, @RNos, x, t);}$
$\mathbf{u = atr(:,:,1); surf(x,t,u), title('Nelin.parabolisksPDV'), xlabel('x'), ylabel('t'))}$
$\mathbf{function\,[c,f,s] = PDV(x,t,u,DuDx) \quad c = 2; f = 4 * u^3 * DuDx, s = 5 * u^3;}$
$\mathbf{function\,[pk,qk,pl,ql] = RNos(xk,uk,xl,ul) \quad pk = uk; pl = ul; qk = 0; ql = 0;}$
$\mathbf{function\ u0 = SNos(x) \quad u0 = x * (1-x);}$
>> **konf_LMB** $\Rightarrow$ iegūstam krāsainu virsmu.

ACTA SOCIETATIS MATHEMATICAE LATVIENSIS
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

MODELLING OF A FLOW OF INCOMPRESSIBLE VISCOUS LIQUID IN A PIPE

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In new technological applications it is important to use vortex distributions in the area for obtaining large velocity fields [1]. The stationary axially- symmetric flow of incompressible liquid in a pipe is modelied. This process is described by the system of the Navier - Sokes equations, where is given on the dimensional form in cylindrical coordinates (r, ϕ, z) [3]:

$$\begin{cases} M(V_z) = -\rho^{-1}\frac{\partial p}{\partial z} + \nu\Delta V_z + F_z \\ M(V_r) = -\rho^{-1}\frac{\partial p}{\partial r} + \nu(\Delta V_r - r^{-2}V_r) + r^{-1}V_\phi^2 + F_r \\ M(V_\phi) + r^{-1}V_r V\phi = \nu(\Delta V_z - r^{-2}V_\phi) \\ \frac{\partial(rV_r)}{\partial r} + \frac{\partial(rV_z)}{\partial z} = 0, \end{cases} \tag{1}$$

Here V_r, V_z, V_ϕ are the radial, axial and azimuthal components of velocity vector V, depending on the coordinates r, z, Δ is Laplace operator, $\Delta g = r^{-1}\frac{\partial}{\partial r}(r\frac{\partial g}{\partial r}) + \frac{\partial^2 g}{\partial z^2}$, F_r, F_z are the components of the external force F, $M(g) = V_r\frac{\partial g}{\partial r} + V_z\frac{\partial g}{\partial z}$ is the convective parts of an equations , ρ, ν are the density and kinematics viscosity ($\eta = \rho\nu$ is the dynamic viscosity), p is pressure.
Eliminating the pressure and using the stream function ψ, the circulation $W = rV_\phi$, the transformed vorticity function $\omega = \omega_\phi/r, (\omega_\phi$ is the vorticity) and different boundary conditions we have for modelling the monotonous finite-difference scheme [2].

The modelling is investigated at different values of parameters Re (Reynolds number), Γ(swirl number) and A(vortex intensity).

The distribution of velocity field for viscous incompressible fluid in a pipe with a system of finite number of circular vortex lines, positioned on the inner surface of the finite cylinder is calculated. Using computer program MAPLE and ANSYS CFX [4] were calculated :
a) Poiseuille flow in a smooth pipe, b) the flow is poured in inside a pipe through a ring, c) the flow is poured in inside a pipe through a circle.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

ONE DIMENSIONAL MATHEMATICAL MODEL OF PLYWOOD PRODUCTION

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The heat mass transfer model in a layered and orthotropic media has been offered in the article [1]. In the article [2] the 3-dimensional heat mass transfer is considered if in addition to the layers there are interlayer. In the article [3] the 3-dymentional heat mass transfer or diffusion with sorption in the orthotropic media with the interlayer is considered. In this article, specifying the generalized theory of the plywood producing [4], we are offering the one dimensional diffusion and sorption problem in the orthotropic media with layers and interlayer as a model for the plywood gluing process, where the interlayer role play the glue layer. The finite difference scheme for solving the ordinary differential equation system is offered.

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ACTA SOCIETATIS MATHEMATICAE LATVIENSIS
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

FUZZY SET THEORY: FOUNDATIONS AND APPLICATIONS

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"More often than not, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership. Clearly, the *class of all real numbers which are much greater than 1*, or *the class of beautiful women*, or *the class of tall men* do not constitute classes or sets in the usual mathematical sense of these terms," wrote L. A. Zadeh in his work "Fuzzy sets" [5] where he first introduced the notion of fuzzy set as "a *class* with continuum of grades of membership".

The aim of this work is to provide a brief introduction to the history of the development of Fuzzy Set Theory. Also we would like to recall the motivation of Fuzzy Set Theory and mention some basic lines of investigation such as

- Fuzzy arithmetic
- Fuzzy measures and integrals
- Fuzzy topology and fuzzy category theory
- Copulas
- Aggregation operators

We continue with the applications of Fuzzy Set Theory. Today, Fuzzy Set Theory provides new impetus for research in the field of Artificial Intelligence [4]. The fuzzy set theory is widely applied in the models of optimal control, decision-making under uncertainty and in other branches.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

ON A POINTWISE EXTENSION OF AGGREGATION OPERATOR

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In this paper we consider an aggregation operator and a pointwise extension of the aggregation operator.

$A : \cup_{n\in N}[0,1]^n \to [0,1]$ is an aggregation operator on the unit interval if the following conditions hold:
(A1) $A(0,...,0) = 0$
(A2) $A(1,...,1) = 1$
(A3) if $(\forall i = \overline{1,n})$ $(x_i \leq y_i)$ then $A(x_1, x_2, ..., x_n) \leq A(y_1, y_2, ..., y_n)$.
(A1) and (A2) are called boundary conditions, and (A3) resembles monotonicity property of A.

If $F(X)$ is the set of all fuzzy subsets of X and $\prec$ is an order defined on $F(X)$ then $\tilde{A} : \cup_{n\in N}F(X) \to F(X)$ is the general aggregation operator with respect to order $\prec$, if the following conditions hold:
($\tilde{A1}$) $\tilde{A}(\tilde{0},...,\tilde{0}) = \tilde{0}$
($\tilde{A2}$) $\tilde{A}(\tilde{1},...,\tilde{1}) = \tilde{1}$
($\tilde{A3}$) $(\forall i = \overline{1,n})$ $(P_i \prec Q_i)$ then $\tilde{A}(P_1,...,P_n) \prec \tilde{A}(Q_1,...,Q_n)$,
where $P_1,...,P_n, Q_1,...,Q_n, \tilde{0}, \tilde{1} \in F(X)$.

$\tilde{A}$ is a pointwise extension of an aggregation operator A if the following holds:
($\tilde{E1}$) $(\forall t \in R$ $\mu_{\tilde{A}(P_1,...,P_n)}(t) = A(\mu_{P_1}(t), ..., \mu_{P_n}(t))$.

We replace monotonicity property (A3) by q-monotonicity property (A4):
(A4) if $(\forall i = \overline{1,n})$ $(\varphi(x_i) \leq \varphi(y_i))$ then $A(\varphi(x_1), \varphi(x_2), ..., \varphi(x_n)) \leq A(\varphi(y_1), \varphi(y_2), ..., \varphi(y_n))$, where
$\varphi : [0,1] \to \{0,1\}$, $\varphi(\alpha) = \begin{cases} 0, \text{ if } \alpha < 1, \\ 1, \text{ if } \alpha = 1. \end{cases}$

Further we study the properties of $\tilde{A}$ with respect to the order $\subseteq_{F_2}$ defined on the set of fuzzy numbers (special type of fuzzy subsets) $F_1(R)$ in the following way: $P, Q \in F_1(R), (P \subseteq_{F_2} Q) \Leftrightarrow (v(P) \leq v(Q))$, where $v(P), v(Q)$ are vertixes of P, Q resp.

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Acta Societatis Mathematicae Latviensis
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TRANSPORT OF CHARGE PARTICLES IN SHOCKED CONDENSED MATTERS; COMPUTATION METHODS AND GRID COMPUTING EXPERIENCE

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This study deals with the modeling of the transfer of charge particles in shocked condensed matters: insulators, semiconductors, metals and liquid electrolytes. The computational method was developed based on the simulation of ion or electron diffusion triggered by the stress-inertial field of a shock wave front. The complete system of electro-diffusion equations is solved numerically.

$$\frac{\partial C_i}{\partial t} = \frac{\partial}{\partial x}\left[D_i\left(\frac{\partial C_i}{\partial x} + \frac{C_i}{k_B T}\frac{\partial}{\partial x}\left(z_i q\varphi - E_{ic}\left(\mu\right) + E_{ik}\left(\mu\right)\right)\right)\right] \tag{1}$$

$$\frac{\partial}{\partial x}\left(\varepsilon\frac{\partial \varphi}{\partial x}\right) = -\frac{q}{\varepsilon_0}\Sigma_{i=1}^{N} z_i C_i \tag{2}$$

$$\mu = \mu_1\left(1 + exp\left(\frac{4}{H}\left(x - \left(D - \nu_1\right)t\right)\right)\right)^{-1} \tag{3}$$

where z_i is the charge number of the particles of the species i; q is the electronic charge; D_i is the appropriate diffusion coefficient; C_i is the concentration of the particles; φ is the electrostatic potential; ε, ε_0 are the dielectric constants; k_B is Boltzman constant; T is the temperature; E_{ic}, E_{ik} are the elastic and kinetic energies per particle defined by the equations for shock wave parameters; μ_1 is the degree of the maximum compression behind the shock front; D and ν_1 are the shock wave and the maximum mass velocities; H is the shock front width.

Full implicit difference scheme with exponential approximation of Eqn.(1)-(2) has been used for the computation. The technology of the usage of grid computation, its advantages and shortages are discussed in details.

The authors have obtained the distribution of the electrostatic potential and electric field over the shocked sample. The full current, the bias and the conduction currents, obtained by the modeling have been compared with the experimental polarization current. Good concurrence of the calculation current with experimental data of shocked KF-electrolyte, silicon and some metals show good working capacity of the models.

Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

TIME-OPTIMAL ADAPTIVE CONTROL SYSTEM WITH MINIMUM OF PERFORMANCE INDEX

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The article deals with the optimal control systems based on minimizing a performance index, such as the integral of the squared error. Our model supposes that the state vector of the system and vector of measurable variables are described by the following equations

$$\begin{aligned} \dot{\mathbf{X}}(t) &= \mathbf{A}(t)\mathbf{X}(t) + \mathbf{B}(t)\mathbf{U}(t) + \mathbf{T}_d(t) \\ \mathbf{Y}(t) &= \mathbf{C}(t)\mathbf{X}(t) + \mathbf{v}(t), \end{aligned} \tag{1}$$

where $\mathbf{A}$ $-$ $(n \times n)$ state matrix, $\mathbf{B}$ $-$ $(m \times r)$ control matrix, $\mathbf{C}$ $-$ $(m \times n)$ matrix of measurable variables, $\mathbf{X}$ $-$ $(n \times 1)$ state vector of the system, $\mathbf{Y}$ $-$ $(m \times 1)$ vector of measurable variables, $\mathbf{U}$ $-$ $(n \times r)$ control vector, $\mathbf{T}_d(t)$, $\mathbf{v}(t)$ - vectors of external actions operating on the object and on the measurable system.

The main task of our investigation is to develop an adaptive control algorithm so that the performance of the system is optimized. The performance of a control system, written in terms of the state variables of a system (1), can be expressed in general as $J = \int\limits_0^{t_f} g(\mathbf{X}, \mathbf{U}, t)\, dt$.

We are interested in minimizing the error of the system; therefore, when the desired state vector is represented as $\mathbf{X}_d = 0$, we are able to consider the error as identically equal to the value of the state vector. That is, we desire the system to be at equilibrium, $\mathbf{X} = \mathbf{X}_d = 0$, and any deviation from equilibrium is considered an error. Therefore, we consider the design of optimal control systems using state variable feedback and error-squared performance indices [1; 2]. The problem of adaptive control of the closed-loop control system based on parametric identification algorithm includes: 1) measurement of the controlled object state and control vectors at each step of controlled coordinates discretization; 2) choice of a proper quality of adaptive control efficiency criterion functional, which is calculated on a specified stage of information update about state and object control vectors; 3) controlled system parameter evaluation (coefficients of state matrix $\mathbf{A}(t)$ and control matrix $\mathbf{B}(t)$) within the applicable time period; 4) formation (correction) of system regulator algorithm subject to computational value of the selected optimal control quality functional and the controlled system parameter evaluation.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 7th Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

MARKOV APPROACH TO NONPARAMETRIC AUTOREGRESSIVE RISK PREDICTION

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The methods and algoritms of time series analysis play an important role in financial econometrics for identification and prediction of risk. The paper deals with the identification and prediction problems of the autoregressive models of nonlinear time series using nonparametric estimates of the conditional mean and conditional variance. One of main problems is development of time series $\{x_t, t \in Z\}$ methods of analysis through regression models without a priori information about the form of dependence of the conditional expected value from its past values (see, for example, [1,2]). Principal reason of waiver of traditional linear models is no Gauss type of random values, describing the dynamics of the real models. We will remind that assumption about normal distribution of time series allows to calculate the conditional expected value of phase variable as linear functional of its past values $\{x_t, s \leq t\}$. We should deal with the estimation of unknown function in nonlinear difference equation of the first order with usual kind of information about the distribution law in many applied problems of regression analysis for time series already in simplest case $x_t = f(x_{t-1}) + h_t$ where h_t are the uncorrelated tailings, on the average equal to the zero. If we designate $\mathcal{F}^t$ minimum sigma-algebra, in relation to which random values $\{h_s, s \leq t\}$ are measured, the needed function can be defined through the conditional expected value $f(x_{t-1}) := \mathbf{E}\{x_t/\mathcal{F}^{t-1}\}$ .To use the sequence of sigma-algebra $\{\mathcal{F}^t, t \in Z\}$ and conditional dispersion $\sigma_t^2 := \mathbf{E}\{h_t^2/\mathcal{F}^{t-1}\}$ tailings h_t can be present in form work of "white noise" $\{\xi_t, t \in Z\}$, (i.e. sequences of the independent identically distributed (i.i.d.) random values with zero mean and by single dispersion) and with conditional standard deviation: $h_t := \sigma_t \xi_t$. For searching for of function $f(x_n)$ we need to create separate discrete intervals of values and then on every interval we can use either least-squares or consider the model of phase space discretization and presentation of values in form eventual number of no splitting areas $\{S_k, k = 1, ..., r\}$ which can be examined as the states of some Markov chain. It is shown that the Markov chain theory can be applied to study nonlinear time series.

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NUMERICAL NODAL BOUNDARY VALUE PROBLEM FOR SOLUTION OF 2-D HELMHOLTZ AND ABSORPTION EQUATIONS

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We consider two 2-D Dirichlet boundary value problems in the region $\bar{G} = G \bigcup \Gamma$:

$$\sum_{n=1}^{2} G_n \partial_{x_n} D_n \partial_{x_n} \Phi(x) + U(x)\,\Phi(x) = 0, \quad x \in G,\, G = \{(x_1, x_2) : 0 < x_n < l_n,\, n = 1, 2\}, \tag{1}$$

$$\gamma_{n_1}(\Phi(x))_{|x_n=0} = \Xi_n(x \backslash x_n),\; n = 1, 2, \quad 0 < x_m < l_m,\, m = 1, 2,\, m \neq n, \tag{2}$$

$$\gamma_{n_2}(\Phi(x))_{|x_n=l_n} = \Psi_n(x \backslash x_n), n = 1, 2, \quad 0 < x_m < l_m,\, m = 1, 2,\, m \neq n, \tag{3}$$

where $G = G(x) > 0$ are Lame coefficients, $D = D(x) \geq d > 0$ are diffusion coefficients and U(x) is the potential and it is either negative or positive.

Because of the potential there are two problems. In case the potential U is positive in the whole region G, in the problem (1) we have Helmholtz equation. And when the potential U is negative in the whole region G, in the problem (1) we have absorption equation.

Schrodinger equation type problem (1-3) can be effectively solved by using nodal method [1], [2]. In this work functional nodal method [3] is used. The method provides continuity of both the flux and the numerical solution, and this is an advantage. Helmholtz and absorption equations are solved numerically by applying the functional nodal method and difference schemes for both equations are constructed in each of the directions, x_1 and x_2. It is shown and proved that each of the schemes for Helmholtz and absorption equations in the directions x_1 and x_2 have the second order of precision, and in addition, the integral precision of schemes is the same.

The article includes description of the algorithm used for acquisition of difference schemes, proof of their order of precision and the results of numerical computations.

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TRIANGULAR NORM BASED OPERATIONS ON L-FUZZY NUMBERS

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This paper deals with a fuzzy analogue of a real number. In the literature on fuzzy mathematics one can find several different schemes for defining fuzzy numbers. We consider the notion originating to B.Hutton's paper [2] and later developed by several authors - see e.g. [3], [4], [5].

Let L be a complete lattice. An L-fuzzy real number is a function $z : \mathbb{R} \to L$ such that

(i) z is non-increasing;

(ii) $\inf_x z(x) = 0_L$, $\sup_x z(x) = 1_L$;

(iii) z is left semi-continuous, i.e. $\inf_{t<x} z(t) = z(x)$.

The set of all fuzzy real numbers is called the fuzzy real line and is denoted by $\mathbb{R}(L)$. The operations of L-fuzzy addition and L-fuzzy multiplication are jointly continuous extensions of addition and multiplication from the real line $\mathbb{R}$ to the L-fuzzy real line $\mathbb{R}(L)$.

The aim of the present paper is to investigate triangular norm based operations on L-fuzzy real numbers. Recall that a triangular norm (T-norm for short) [1] is an associative, commutative binary operation on the lattice L which is non-decreasing in each argument and has the neutral element 1_L. In the case when $L = [0, 1]$ most of our attention is confined to three T-norms:

(i) the minimum T-norm $T_M(x, y) = min\{x, y\}$;

(ii) the product T-norm $T_P(x, y) = xy$;

(iii) the Lukasiewicz T-norm $T_L(x, y) = max\{0, x + y - 1\}$.

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COINS PROBLEM

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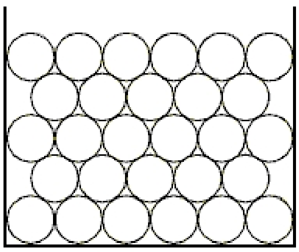
In a combinatorial geometry and other fields the following problem is frequently considered: figures F and G are given. Find out, how many figures G we can insert in the figure F such that they will not overlap and will not go beyond the borders of the figure F. In this work we consider the following problem: a square-shaped box is given, the side length of which is n ($n \in \mathbb{N}$), and an unbounded amount of round coins whose diameter is 1. We need to find out what is the maximal number of coins X_n which we can insert in this box (such that they do not overlap and will not go beyond the borders of the box). From now on we assume that coins are rings and the box is a square. In the work we will show several packing algorithms of rings, we will investigate the optimality of these algorithms, and the quantity X_n will be estimated. For small n, exact values of X_n will be shown.



hexagonal packing

Fig. 1.

We must notice that, solving this problem one can think that an effective algorithm for arranging rings is to lay out them in a hexagonal packing, see Figure 1 and [1]. It means that, first, we arrange a line with n rings, then the line with $n-1$ rings, etc. The number of rings we can arrange in this way we denote with $\mathrm{H}(n)$. In this work we show that for any natural n between numbers n and $n+2$ one can always find such k that this packing is not optimal. It means that for this k $\mathrm{H}(k) < X_k$. We also show that, in fact, for any natural m there exists such $n \leq 11m$ that $X_n - \mathrm{H}(n) \geq m$.

The best lower bounds for X_n that the author has obtained for $n = 1, 2, \ldots, 7$ are n^2. For $n = 8, 9, \ldots, 25$

these bounds are given in the table below.

n		n		n	
8	68	14	216	20	436
9	86	15	247	21	492
10	106	16	280	22	538
11	128	17	315	23	586
12	152	18	352	24	636
13	179	19	392	25	690

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Acta Societatis Mathematicae Latviensis
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STOCHASTIC REGULARIZATION OF SINGULAR TYPE LYAPUNOV EQUATION FOR MARKOV EVOLUTIONS.

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A dynamical system this paper deal with consists of radial motion $\frac{dx^\varepsilon}{ds} = f(x^\varepsilon,\, y^\varepsilon + \omega\tau^\varepsilon,\, \xi^\varepsilon(s),\, \tau^\varepsilon)$ and rotation $\frac{dz^\varepsilon}{ds} = g(x^\varepsilon,\, y^\varepsilon + \omega\tau^\varepsilon,\, \xi^\varepsilon(s),\, \tau^\varepsilon)$ defined by differential equations in $\mathbb{R}^n \times \mathbf{\Phi}$ where $\tau^\varepsilon := \frac{s}{\varepsilon}$, $\mathbf{\Phi}$ is connected closed manifold in $\mathbb{R}^m$, and $\xi(t)$ – the right-continuous homogeneous Feller-Markov process [1] on probability space $(\Omega, \Im, P)$ with continuous C-infinitesimal operator $\frac{1}{\varepsilon}Q$ and compact phase space Ξ. As it has been proved in [2] for asymptotic stability analysis of radial motion one may apply the second Lyapunov method with Lyapunov equation in a form

$$\begin{aligned}(\mathbf{L}^\varepsilon v)(x,y,\xi,\tau) &= \frac{1}{\varepsilon}\Big\{\frac{\partial}{\partial\tau} + Q\Big\}v(x,y,\xi,\tau) + \big\{(f(x,y+\omega\tau,\xi,\tau),\nabla_x) \\ &+ (g(x,y+\omega\tau,\xi,\tau),\nabla_y)\big\}v(x,y,\xi,\tau)\end{aligned}$$

Evidently this equation is of singular type as $\varepsilon \to 0$ and therefore to analyse the above Lyapunov equation for sufficiently small positive ε one need some regularization procedure. Under assumptions that process $\xi(t)$ is ergodic with unique invariant probability measure $\mu \in C^*(\Xi)$ and for any $v \in \mathbf{C}(\Xi)$ and $s > t$ the conditional mathematical expectation of $v(\xi(s))$ under condition $\xi(t) = \xi$ exponentially tends to number $\hat{v} = \int_\Xi v(\xi)\mu(d\xi)$, that is $\mathbb{E}_{t,\xi}\,|\{v(\xi(s)) - \hat{v}\}| \le \|v - \hat{v}\| \exp\{-\rho(s-t)\}$ one may define on the space $\mathbf{C}(\Xi, \mathbb{R})$ *regularizing operator*

$$(\mathcal{R}u)(\xi,t) := \int_t^\infty [(E_{t,\xi}u)(\xi,s) - \hat{u}(s)]ds + \int_0^t [\hat{u}(s) - \tilde{u}]ds$$

which permits to chose Lyapunov function in a following form

$$v(x,y,\xi,\tau) := v_0(x,y) + \varepsilon\{(\mathcal{R}f(x,y,\xi,\tau),\nabla_x) + (\mathcal{R}g(x,y,\xi,\tau),\nabla_y)\}v_0(x,y)$$

and to deal with this function for further stability analysis of radial motion.
Proposal method and algorithm are illustrating by analysis of stochastic parametrical resonance problem [3] in linear oscillator

$$\ddot{x} + \delta\dot{x} + (\omega^2 + h\cos(\nu t + \xi(t)))x = 0$$

with small of dissipativity δ and swapping h parametrs, and fast oscillating Markov process $\xi(t)$ given on unit circle $\Xi = \mathbb{S}^1$.

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Acta Societatis Mathematicae Latviensis
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HOMOGENIZATION OF SOME STRUCTURES WITH DISCONTINUOUS SOLUTIONS

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We consider an analogue of the situation described in [1] where for some particle diffusion processes boundary conditions on the interface between two materials are of the type

$$k_1c_1 = k_2c_2, \qquad D_1\nabla c_1 = D_2\nabla c_2, \tag{1}$$

where indexes 1 and 2 correspond to the 1st and the 2nd material, c_1 and c_2 are concentrations of particles, D_1 and D_2 are the standard diffusion coefficients, but k_1 and k_2 reflect the velocities of particles in the 1st and the 2nd material respectively. We show that , under some assumptions on the structure and the limit behaviour of the layout of materials, this situation can be treated within the framework of standard G- convergence and homogenization theory.

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CONJUGACY OF DIFFERENCE EQUATIONS IN THE NEIGHBOURHOOD OF INVARIANT MANIFOLD AT BANACH SPACE

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In Banach space $\mathbf{X} \times \mathbf{E}$ the system of difference equations

$$\begin{aligned} x(t+1) &= g(x(t)) + G(x(t), p(t)), \\ p(t+1) &= A(x(t))p(t) + \Phi(x(t), p(t)) \end{aligned} \tag{1}$$

is considered. Sufficient conditions under which there is an Lipschitzian invariant manifold $u\colon \mathbf{X} \to \mathbf{E}$ are obtained. Using this result we find sufficient conditions of conjugacy (1) and

$$\begin{aligned} x(t+1) &= g(x(t)) + G(x(t), u(x(t)), \\ p(t+1) &= A(x(t))p(t). \end{aligned} \tag{2}$$

The second system splits into two parts. The first of them does not contain the variable $p \in \mathbf{E}$, while the second one is linear in respect to $p \in \mathbf{E}$. Relevant results concerning partial decoupling and simplifying of the noninvertible difference equations are given also.

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L-FUZZY VALUED T-MEASURE OF L-SETS[1]

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Let X be a nonempty set, L a completely distributive lattice, $F(X,L)$ the set of all L-fuzzy subsets of X. The operations of L-sets are defined by using a triangular norm T, it's corresponding triangular conorm and an involution.

We consider such T-norm based classes of L-sets as T-semirings and T-tribes [1], such L-fuzzy valued functions as elementary T-measures, exterior T-measures and T-measures. For our purposes we decided in favour of the L-fuzzy real numbers, as they were first defined by B.Hutton [2].

The aim of the present work is to construct an L-fuzzy valued T-measure of L-sets by extension a given crisp finite measure ν defined on a sigmaalgebra Φ of crisp subsets of X. We generalize the well known construction of the classic measure theory to the L-fuzzy case.

In order to do this we describe a T-semiring and an elementary T-measure defined on this T-semiring on the base of Φ and ν. Then we extend the elementary T-measure to the exterior T-measure defined on $F(X,L)$ and obtain the T-tribe Σ of measurable L-sets. To get the L-fuzzy valued T-measure we restrict the exterior T-measure to Σ.

This construction for the case of the minimum T-norm and $L=[0,1]$ was considered in [3] and for the case of a distributive T-norm was presented in [4].

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ON NONLINEAR SPECTRA

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We study the second order boundary value problem

$$-x'' = \mu x^+ - \lambda x^-, \quad x^{\pm} = \max\{\pm x, 0\}, \tag{1}$$

$$x(0) = 0, \quad \int_0^1 x(s)ds = c, \quad |x'(0)| = 1, \quad c \in \mathbb{R}. \tag{2}$$

By the Fučík spectrum we mean the set

$$\{(\mu, \lambda) \in \mathbb{R}^2 : \text{ the problem (1), (2) has a nontrivial solution}\}.$$

We describe properties of the Fučík spectrum and state that: the Fučík spectrum for the problem (1), (2) exists for each c except $c = \pm\frac{1}{2}$. Moreover the following properties hold:

- if $c \in (-\infty; -\frac{1}{2})$ then the spectrum consists of a unique branch F_{0-}^-;
- if $c \in (-\frac{1}{2}; -\frac{2}{\pi^2}]$ then the spectrum consists of a unique branch F_{0+}^-;
- if $c \in (-\frac{2}{\pi^2}; \frac{2}{\pi^2})$ then the spectrum consists of the branches F_i^- and F_i^+, $i = 1, 2, \ldots$;
- if $c \in [\frac{2}{\pi^2}; \frac{1}{2})$ then the spectrum consists of a unique branch F_{0+}^+;
- if $c \in (\frac{1}{2}; +\infty)$ then the spectrum consists of a unique branch F_{0-}^+.

We also get explicit formulas for the spectrum of this problem.

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OSCILLATORY PROPERTIES OF LINEAR THIRD-ORDER DIFFERENTIAL EQUATIONS

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Oscillatory behavior of solutions of a third-order linear ordinary differential equation

$$x''' + p(t)x'' + q(t)x' + r(t)x = 0 \tag{1}$$

is discussed. A connection between double zeros of solutions and vanishing of the minor of the Wronskian is established. Also, a geometrical interpretation of this fact is given.

In the study of the double zeros adjoint equation plays an important role. Adjoint equation and equation which has similar properties concerning distribution of zeros are considered.

Two classes of the third-order equations are considered. Examples of equations of these classes are considered which exhibit certain properties similar to properties of solutions to the second-order equations.

Illustrative examples and figures are provided.

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VARIABLE-BASIS SOBRIETY AND SPATIALITY [1]

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We continue our investigation of the classical Papert-Papert-Isbell adjunction between the categories of topological spaces and locales. The relevance of the adjunction is manifold. On the one hand, it provides an appropriate environment in which to develop topology. On the other hand, the notions of sobriety and spatiality give a powerful tool for deriving various representation theorems. In a series of papers U. Höhle, A. Pultr and S. E. Rodabaugh (see, e.g., [1; 2] and the references therein) considered fuzzy analogues of the adjunction replacing locales by either localic semi-quantales or by L-locales over a complete chain L. These investigations not only generalized the classical theory, but in some cases uniquely streamlined it via two explicit evaluation maps. In particular, S. E. Rodabaugh postulated a meta-theorem saying that the explicit form of an evaluation map is obtained iff the poslat philosophy of fuzzy sets (that a subset be replaced with a mapping into a lattice) is adopted.

It is important to note that most of the above-mentioned fuzzifications use varieties of algebraic structures which are *ordered* and have *everywhere* defined operations. In [4] we extended the meta-theorem to non-ordered situations considering an arbitrary variety of algebras. In [3] we used the category of Q-algebroids over a unital commutative quantale Q to extend the meta-theorem to partial algebras. The results of both papers are fixed-basis. This talk provides a variable-basis generalization of [4] as follows (it is an open question whether our approach is the only one).

Let $\mathbf{A}$ be a variety of algebras and let $\mathbf{LoA}$ be the dual of $\mathbf{A}$. The category $\mathbf{LoA}$-$\mathbf{Top}$ is defined by the following data. The objects are triples $(X, A, O_A(X))$, where $A \in \mathbf{LoA}$, $X \in \mathbf{Set}$ and $O_A(X)$ is a subalgebra of A^X. The morphisms $(f, \varphi) : (X, A, O_A(X)) \to (Y, B, O_B(Y))$ are $\mathbf{Set} \times \mathbf{LoA}$-morphisms $(f, \varphi) : (X, A) \to (Y, B)$ such that $((f, \varphi)^{\leftarrow})^{op}(p) := \varphi^{op} \circ p \circ f \in O_A(X)$ for every $p \in O_B(Y)$. Let $\mathbf{TOP}$ (resp. $\mathbf{LOALG}$) be the subcategory of $\mathbf{LoA}$-$\mathbf{Top}$ (resp. $\mathbf{LoA} \times \mathbf{LoA}$) with morphisms all pairs (f, φ) (resp. (φ, ψ)) such that φ is an isomorphism. There exists a functor $O : \mathbf{TOP} \to \mathbf{LOALG}$ given by $O(X, A, O_A(X)) := (A, O_A(X))$ and $O(f, \varphi) := (\varphi, (f, \varphi)^{\leftarrow})$.

Theorem. *The functor $O : \mathbf{TOP} \to \mathbf{LOALG}$ has a right adjoint.*

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L-FUZZY APPROXIMATIVE SYSTEMS AND L-FUZZY APPROXIMATIVE SPACES

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Let L be a fixed complete residuated lattice and let X be a set. By an upper L-fuzzy approximation operator on X we call a mapping $u : L^X \to L^X$ such that

1. $u(0_X) = 0_X$;
2. $A \le u(A)\ \forall A \in L^X$;
3. if $A \le B$ then $u(A) \le u(B)\ \forall A, B \in L^X$;
4. $u(A \vee B) = u(A) \vee u(B)$.

By a lower L-fuzzy approximation operator on X we call a mapping $l : L^X \to L^X$ such that

1. $l(1_X) = 1_X$;
2. $A \le l(A)\ \forall A \in L^X$;
3. if $A \le B$ then $l(A) \le l(B)\ \forall A, B \in L^X$;
4. $l(A \wedge B) = l(A) \wedge l(B)$.

Let l and u be a lower and an upper L-fuzzy approximative systems on a set X. Then the triple (L, l, u) will be refered to as an L-fuzzy approximative system on a set X, and the corresponding quadruple (X, L, l, u) will be called an L-fuzzy approximative space.

In the talk we shall discuss some properties of such spaces, In particular, their relations to L-topologies in the sense of Chang-Goguen, [1], [2], L-topologies in the sense of [3], rough sets as they were defined by Z. Pawlak [4], and some othe structures on a given setwill be considered.

Basic properties of the category LFAS of L-fuzzy approximative spaces and appropriately defined morphisms between such spaces will be also discussed.

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LIMIT THEOREM FOR EUROPEAN OPTION HEDGING IN NO COMPLETE SECURITY MARKET

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There are two main approaches to security market modelling. The first earliest model proposed by J. Cox and his following (see description in detail in [1]) makes good use of discrete time and binomial values of securities. This simplification permits authors to find call option fair price in a compete form which is handy shape for valuation of financial risk. But this approach is ill-formed for analysis of no complete market when stock interest rate has more then two possible states. In this case one can apply continuous market model with lognormal distribution of stock price dynamics (see [3] and references there) described by a system of stochastic differential equations. In this paper we will discuss an accuracy of the above diffusion approximation for the simplest model of discrete stock market with the most familiar type of option which is the option to buy a stock at a given price at a given time. Our mathematical model have a form of two linear difference equations for stock and European option portfolio dynamics with random coefficients sensitive of interest rate term structure. Under assumptions that one can model the above interest rate process as sequence of independent identically distributed random variables with finite number of states we derived a formula for fair price of option hedging as conditional expectation of payoff at maturity function based on either of martingale measures. Next applying well known results (see [2] and references there) relative to convergence of Markov difference equations to stochastic Ito differential equations we are comparing the above results to classical Black-Scholes formula (see [4] and references there) derived for the above equations.

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ON EXTENDING L-VALUED RELATIONS TO THE L-POWERSETS

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Let X be a set L be a complete bounded lattice and $R : X \times X \to L$ be an L-valued relation on X. Given $A, C \in L^X$ we set

$$\mathcal{R}(A, C) = \bigwedge_{x,z \in X} \big(R(x, z) * A(x) \mapsto C(z)\big).$$

Thus we obtain an L-valued relation

$$\mathcal{R} : L^X \times L^X \to L.$$

In our previous work [1] we used this relation in order to extend an L-valued equality from X to L^X. In the present talk we shall discuss how different properties of a given relation R are reflected in the properties of the relation $\mathcal{R} : L^X \times L^X \mapsto L$ and in the properties of its restriction to the subfamily L^X_R of all R-extensional L-subsets of X.

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CONFIDENCE BANDS FOR STRUCTURAL RELATIONSHIP MODELS

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Consider the two-sample problem, where i.i.d. random variables $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ are independent and have some unknown distribution functions F_1 and F_2 respectively. Well known and important in statistics is the simple location model, which assumes that the two distribution functions differ from each another by a shift, i.e.

$$F_1(t) = F_2(t - \mu)$$

for all $t \in \mathbb{R}$, where μ is some positive constant. This model reveals some "relationship" between F_1 and F_2. Recently Freitag and Munk [2] have introduced the notion of *structural relationship models*, which has the following general form

$$F_1(t) = \phi_2^-(F_2(\phi_1^-(t,h)),h), \quad t \in \mathbb{R},$$

where ϕ_1, ϕ_2 are some real-valued functions, ϕ_i^- denotes the inverse function with respect to the first argument, $h \in \mathcal{H} \subseteq \mathbb{R}^l$ is the "structural parameter", l is an integer. This general object describes also the well-known location-scale and Lehman's alternative model.

To construct a two-sample goodness-of-fit test for structural relationship model one possibility is to construct simultaneous confidence bands for P-P or Q-Q plot for such structural relationship models. This has been done in PhD theses of Valeinis [6]. First, one has to estimate the unknown structural parameter h, which can be done using some distance functional (we use Mallow's distance). Second, to construct the confidence bands, we use the empirical likelihood method introduced by Owen [4], [5].

Recently Hjort et al. [3] introduced the plug-in empirical likelihood method for one sample case. To construct the confidence bands for structural relationship models the general plug-in empirical likelihood method for the two-sample case has been established in Valeinis [6].

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Abstracts of the 7$^{\text{th}}$ Latvian Mathematical Conference, April 18–19, 2008, Rēzekne, Latvia

TYPES OF SOLUTIONS TO BOUNDARY VALUE PROBLEMS FOR Φ-LAPLACIAN EQUATION

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Boundary value problem for Φ-Laplacian equation

$$\frac{d}{dt}(\Phi(t,x')) + f(t,x) = 0, \qquad t \in I := [a,\, b] \tag{1}$$

together with the boundary conditions

$$\begin{cases} x(a)\cos\alpha - \Phi(a,x'(a))\sin\alpha = 0, \\ x(b)\cos\beta - \Phi(b,x'(b))\sin\beta = 0 \end{cases} \qquad 0 \le \alpha < \pi, \quad 0 < \beta \le \pi \tag{2}$$

is considered, where functions Φ and f are continuous and Lipschitzian with respect to x' and x respectively.

We reduce problem (1), (2) to a problem

$$\begin{cases} x' = \Phi^{-1}(t,\, y), \\ y' = -f(t,x) \end{cases} \tag{3}$$

with the boundary conditions

$$\begin{cases} x(a)\cos\alpha - y(a)\sin\alpha = 0, \\ x(b)\cos\beta - y(b)\sin\beta = 0. \end{cases} \tag{4}$$

The quasilinearization method is applied to obtain multiplicity results.

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Acta Societatis Mathematicae Latviensis
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COMPARISON OF STABILITY CONDITIONS FOR SEMI-IMPLICIT DIFFERENCE SCHEMES IN SOLUTION OF ADR EQUATION

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Modern geochemical transport models become more and more complex in last years. Therefore, for rapid development of computational technologies, capacity of computational speed is not satisfactory in lots of cases. Volume of calculations in mathematical models for geochemical transport, described by advection-diffusion-reaction (ADR) equation, is also too magnificent for the majority of modern computers. One of the most frequent reason, what causes difficulties in mathematical calculations, is the restriction for the time step, raised from numerical instability. Decreasing time step causes uncontrollable growth of computational time, required for solution of ADR equation. One of the reasons of searching for new numerical methods, regardless of all known numerical methods [1], [2], [3], is the attempt to make the stability conditions better. Such new method- the propagator method- is considered in this work, results of stability conditions are compared to semi-implicit central difference scheme. The semi-implicit propagator difference scheme depends of parameter values, can be absolutely stable, or in comparison to traditionally used semi-implicit central difference scheme, restrictions for time step are weaker. Even better, when the scheme tends to stationary solution, the propagator scheme becomes absolutely stable. It should be noted, that there are stability conditions for fully implicit and explicit difference scheme [2], [3], [4] described in literature. For semi-implicit difference scheme, where the Laplacian is defined from current time layer $l+1$, advective and reactive terms are coming from previous time layer l, stability conditions are not described. In this work, by using of von Neumann method, strict stability conditions for both, semi- implicit central difference scheme, and semi- implicit propagator difference scheme are obtained. Stability criterions for time step restriction were used for selection of time step when solving ADR-equation with semi-implicit propagator difference and central difference scheme. For both difference schemes criterions for time step, obtained by theoretical calculations, showed good compliance to numerical experiment. For the semi-implicit propagator scheme in the process of calculation the time step can be increased step by step, until it becomes absolutely stable. It is shown that use of propagator scheme is more effective when use of semi-implicit central difference scheme. This work is made with the support of the ESF, project 2004/0001/VCD1/ESF/PIAA/04/NP/3.2.3.1/0001/0001/0063.

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ACTA SOCIETATIS MATHEMATICAE LATVIENSIS
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COMBINATORIAL MAP AS MULTIPLICATION OF COMBINATORIAL KNOTS

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Combinatorial map theory seems to appear more important branch of mathematics than was thought earlier [1; 2; 5]. In [3] was shown that each normalized map may be expressed as $P = \mu \cdot \alpha$, where μ is combinatorial knot of the map, and α is called *knotting* and it is selfconjugate map in sense that $\alpha^{\pi} = \alpha$, where π is map's inner edge rotation.

In [3] was shown that particular choice of edge rotation ρ by fixed π induces partitioning of the set C of elements maps are acting on, into to subsets C_1 and C_2 [in general in several ways] so that the *knot* $\mu = \begin{cases} C_1 : & \pi \\ C_2 : & \rho \end{cases}$ is defined. Here, knot μ as permutation has 2^k choices if k is number of cycles in it. ρ with choice of particular μ partitions π into $\pi_1 \cdot \pi_2$, where we call π_1 *cut edges* and π_2 *cycle edges*, so that $P \cdot \pi_1 : C_1 \mapsto C_2$ and $P \cdot \pi_2 : C_1 \mapsto C_1$. In [4] was shown that by fixing μ map P may be expressed as multiplication $\gamma_1 \cdot \gamma_2 \cdot \pi_2$, where γ_1 acts within C_1 and γ_2 acts within C_2.

In [4] we got formulas for μ and α, i.e., $\mu = \gamma_2 \pi \gamma_1^{-1}$ and $\alpha = \gamma_1 \gamma_1^{\pi}$.

THEOREM 1. *$\rho \cdot \pi$ [or $\pi \cdot \rho$] is equal to some combinatorial knot μ squared and one or other color cycles induced from this knot reversed.*

THEOREM 2. *$\mu \cdot \pi$ contains squared knot's cycles of only one color. For vertex rotation $\mu \cdot \pi$ corresponding face rotation and knot are equal to μ, and knotting equal to π. $\gamma_1 = id$, $\gamma_2 = \mu \cdot \pi$, and $\pi_1 = \pi$.*

THEOREM 3. *$\gamma_1 \gamma_1^{\pi}$ is some knot's square. Let denote this knot ν.*

THEOREM 4. *Every combinatorial map P can be expressed as multiplication of knots in the form $P = \mu \cdot \nu^2 \cdot \pi_1$.*

ν we call map's inner knot. Truly, π_1 may be consider as knot, called trivial knot, and P is expressed as multiplication of knots.

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FINDING POINTS OF A POINTLESS TOPOLOGY

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The notions presented in the following definitions are very close to each other (though those are usually studied using dramatically different tools): all of them are kind of a topological structure.

Definition 1. Given a nonempty set X. A subfamily $\mathcal{T} \subseteq 2^X$ is a *topology* provided:
1. $\varnothing, X \in \mathcal{T}$,
2. $\bigcup \mathcal{U} \in \mathcal{T}$ holds for every $\mathcal{U} \subseteq \mathcal{T}$,
3. $\bigcap \mathcal{V} \in \mathcal{T}$ holds for every finite $\mathcal{V} \subseteq \mathcal{T}$.

Definition 2. Given a set X, an ideal $\mathcal{I} \subseteq 2^X$. A subfamily $\mathcal{T} \subseteq 2^X$ is an *i-topology* [2; 3] provided:
1. $\varnothing, X \in \mathcal{T}$,
2. for every $\mathcal{U} \subseteq \mathcal{T}$ there exists $U \in \mathcal{T}$ such that $\bigcup \mathcal{U} \Delta U$ lies in $\mathcal{I}$,
3. for every finite $\mathcal{V} \subseteq \mathcal{T}$ there exists $V \in \mathcal{T}$ such that $\bigcap \mathcal{V} \Delta V$ lies in $\mathcal{I}$,
4. $\mathcal{T} \cap \mathcal{I} = \{\varnothing\}$.

Definition 3. An object in $\mathbf{Frm}^{\mathbf{op}}$ is called a *locale* [1]. By $\mathbf{Frm}^{\mathbf{op}}$ is denoted the category of frames where objects are complete infinitely distributive lattices and morphisms are maps preserving arbitrary joins and finite meets.

The central idea of locale theory is to study topological properties without referring to points, that is why locales are commonly known as *pointless topologies*. Nevertheless, the definition of *the points of a locale* [1] (the homomorphisms acting from $\mathbf{2}$ to a given lattice) appears almost every time locales are mentioned even though not every locale is *spatial*, that is has enough points [1]. Looking that way, it is interesting to notice that every frame F forms an i-topological space [2] that may have much more points than those defined by $\mathrm{pt}\colon \mathbf{2} \to F$.

We study similarities and differences between topologies, i-topologies and locales using the tool of *finding points of a pointless topology*, t.i. considering a locale as an i-topology which, in its turn, is a generalization of a classical topological space [3].

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INDEX